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Transcript for October 23- 25, 2006

"Developing an Algorithm for Multiplying Fractions:
Bits and Pieces II, Using Fraction Operations, Investigation 3"

The class is seen working on Investigation 3.3,
"Modeling More Multiplication Situations"

The video was shot in real time and edited from 2 days,
approximately 2 hours, to 22 minutes.

Bits & Pieces II, Investigation 3
Class: 6th Grade
Date: October 23-27, 2006

Chapter 1: Launching 3.3.

Approximate time: 00 – 04:05 (Times from start of video)

Slide:

Day 4 begins with questions from the previous night's ACE assignment. Students derive a geometric way to understand why multiplication of fractions (like 2/3 of ¾) is commutative. Then Teri launches into 3.3 with a long discussion about the problems in the Getting Ready.

Slide

Getting Ready problems.

Slide

In this introductory discussion students suggest different strategies that will eventually all be useful in multiplying mixed numbers, though they are not all accepted by the class at this point.

[2-1/2 x 4/7]

Line 1, 00:22 T: Kristen, do you have any ideas about how to estimate that one?

S11 [Kristen]: Well, um, I think it might be, um, well, 'cause if it was four, um, four sevenths, um, times two and one half it would be like four sevenths of two and a half --

T: Okay. So you're thinking about it like this?

S11(Kristen): Yeah, so --

T: Okay.

S11(Kristen): Four sevenths is about half so I think it would be

Line 10, 00:45 like two.

T: So you're thinking of half of two and a half?

S11(Kristen): Yeah.

T: So what would a half of two and a half be?

S11: Oh, wait, it would be, um, one and one fourth.

T: So in this case we're multiplying. So can we flip-flop it or no?

Class: No.

T: We can't? Why not? Why can't we?

S__ (Nikki): 'Cause we

Line 20, 1:08 T: Somebody tell me why we can't. [Teacher calls name of student with raised hand].

S20 [Nikki]: 'Cause we're using a mixed number.

T: So how does that change it?

S20(Nikki): 'Cause if you change it then you have more than a whole when you just started out less than a whole.

T: Hmm. Kaylie, what do you think?

S6: To make it less confusing, maybe you could change the two wholes into halves. Like, for, five halves --

T: Oh.

Line 30, 1:27 S6: So maybe --

T: Would that help us? What if we called this five halves of four sevenths?

Class: [Unintelligible responses]

T: But could you multiply that using our algorithm?

Class: Yeah.

T: Let's do it just to see what we get. Could we do five halves of four sevenths? What would we end up with?

Class: Fourteen twentieths, twenty fourteenths.

T: Actually, twenty fourteenths, right? And what is that

Line 4, 1:56 the same as?

Class: One whole and six fourteenths.

T: One whole and six fourteenths. Hmmm. Is that about, now these are estimates. Is that close to this estimate?••

Class: Yes.

T: No.

Class: Not really.

T: No. Isn't it about, almost one and a half, and that's one and a fourth.

T: Nope. Not even close… I tell you what. Let's look at

Line 50, 2:25 one more and let's see if we can just figure out what it's saying, not necessarily how to solve it. But what is it saying if I have three and a fourth times two and eleven twelfths. What does that mean? What does than mean? I need to get some more of you involved. Michaela, what does it mean now?

S21 [Michaela]: Three and one fourth groups of two and eleven twelfths.

T: Say that again, Michaela. What does it mean?

S21(Michaela): Three and one fourth groups of two and eleven

Line 60, 2:46 twelfths.

S__ [unidentified]: Maybe we have to add --

T: Hold on for just one second. Let's look at this for a minute. Does this just mean I have more than three groups of more than two?

Class: Yeah. Um huh.

T: Let that sink in for a second. Is it possible to figure that out?

Class: Yes, yeah.

T: Okay. Now, go ahead, you were going to say something.

Line 70, 3:17 S11 [Kristen]: Maybe we have to add two and eleven twelfths like three times, and then another fourth time.

T: What, what do you guys think about that?

T: You're gonna have to think hard about how can I model this and figure out how to solve this. I'm not so interested in an algorithm today. I'm interested in how can we literally solve through a drawing or something a problem like this? How can we do it? And it looks like some of you have some good ideas on ways to start. Maybe you're gonna have a picture of something and be fractioning off parts or

Line 80, 3:55 whatever. We're not on some mission to find an algorithm. We're on a mission to make sense of what is this really saying and how can we do that? How could we find three and a

quarter groups of two and eleven twelfths?

Chapter 2: Exploring 3.3

Approximate time: 04:05 – 5:06 (Times from start of video)

[2/3 of 16]

Line 1,4:06 T: You know what. Try your idea. You try that and see what you get. And you're trying something else.

S6(Kaylie): Uh huh.

T: What are you trying?

S6(Kaylie): Uh, splitting each ounce into thirds.

T: And then what are you going to do?

S6(Kaylie): Color in two thirds of each.

T: Of each ounce?

S6(Kaylie): Yeah.

Line 10, 4:22 T: You try that and see how yours, 'cause your strategy is different from hers, see how yours compares with what she's doing.

T: So you took a two thirds of each of those ounces?

S22 [Justin]: Yup.

T: So now how many thirds do you have altogether that you've colored in?

S22(Justin): *[Student counts]* Thirty two.

T: Thirty two thirds?

S22(Justin): [Nods affirmatively]

Line 20, 4:43 T: So how much is that? I want to see if you got the same answer as Caleb did. Let's write it down just so we can see, okay. Can we switch that over to a mixed number? How much would that be?

T: Did you? Oh, you did yours similar. I'm going to get you guys to put, since you almost had the same exact idea she had, would you guys put your work for C on there?

Slide

The class ends with many students feeling this was quite a challenge, and some groups having reached solutions they are going to present to their peers the next day. Teri assigns them some practice on something they are comfortable with.

Chapter 3: Continuing to Explore 3.3.

Approximate time: 05:06 – 07:37 (Times from start of video)

Slide

Day 5 begins with students continuing to make sense of multiplying mixed

numbers and getting their strategies on to posters to share.

(3.3D, 10 1/2 x 2 1/3)

[Miscellaneous classroom background noises and voices; student writing division problem]

Line 1, 5:25 S9 (Becca): Two and one third, two and one third, and then you add'em all up? But what would you do for the half? We need to figure out the half day.

T: Can you do this in a whole day, so how could you figure out what you do in half of a day?

S9(Becca): Would you split it in half?

T: Does that make sense to you?

S9(Becca): Yeah.

T: What do you think that they can accomplish? If they

Line 10, 5:52 could do two acres in a whole day, what do you think they could do in half a day?

S9(Becca): Half of two and one third?

T: Could you figure that out?

S9(Becca): It would be one and one sixth --

T: Can I see?

S9(Becca): Or one and one third.

S__ (student off camera): You guys, I figured it out. The teacher said, the teacher said that --

S5 and S6 (Ellie and Kaylie)*[having discussion]:* [Unintelligible].

Line 20,6:16 S5(Ellie): Minus half of it off.

S6(Kaylie): Do I need to do another day?

S5(Ellie): Kind of. I think [unintelligible] two and one third, you have to break that in half, divide it by two. And what would that get you?

S6(Kaylie): Huh?

S5(Ellie): You have two and one third on a regular day, but --

S6(Kaylie): Yeah.

S5(Ellie): Today you only have it for half amount of time, that means you only get say half amount of crop, so if you

Line 30, 6:48 divide this by two what would that get you? Would two divided, two -

S6(Kaylie): Why do you have to divide it in half?

S5(Ellie): Because you only have half the time, that means you only harvest half the amount.

S6(Kaylie): I'm still confused.

S5(Ellie): Two divided by two, what's that equal?

S6(Kaylie): But why are we doing two divided by two?

S5(Ellie): Because you only have half the amount, like, you have, I agree with this part, but then you have to add this half

Line 40, 7:17 on. You only have this part right here. It's the only part you have, okay?

S6(Kaylie): Yeah.

S5(Ellie): So, what, now, now, see you only have half a day

so you can't have, you can't harvest the same amount as you would do in a regular day. So, if you usually harvest two and one third --

S6(Kaylie): Oh, I get it.

S5(Ellie): So what would that equal?

S6(Kaylie): One and one sixth.

Chapter 4: Summarizing 3.3B

Approximate time 07:37 – 09:37 (Times from start of video)

[3.3B, 2/3 of 16]

Line 1, 7:38 T: Could, could the purple drawing right on the left go next? Is yours similar or different from what those guys just talked about?

S__ (off camera): Similar:

T: Okay. Talk about what you guys did.

S3 and S22 (Justin and Drew) *at board.*

S22: Well, first we drew like sixteen wholes, like this is one whole, and then we split each whole into thirds, and then we colored in two of the thirds, and then we had to

Line 10, 8:13 add all those up to get our answer.

T: Similar kind of thing?

Class: Yeah.

T: Okay. Alright. Thanks, guys. So, if I want to look at this - is it okay if I write on one of your drawings? Is it okay if I write here 'cause there's some room here? Is it okay? Okay. So when I heard you say two thirds of sixteen, you ended up with thirty two thirds. Yes? Yeah?

S11 [Kristen]: It looks like, um, it was sixteen times two. Sixteen times two is thirty two.

Line 20, 8:45 T: Oh.

S__(Taylor): Three times sixteen is thirty __

T: Three times sixteen is three?

S16 (Jesse): Oh, the denominator stays the same. The numerator --

T: Hmmm. Did you guys notice that?

Class: Yeah. Yes, I just noticed that.

T: What?

S5 (Ellie): It would be sixteen over one because --

T: I can write sixteen as sixteen over one?

Line 30, 9:08 S5(Ellie): Yeah, because three times one is three and then two times sixteen is thirty two.

•Class: Yeah, oh.

S5(Ellie): 'Cause you have sixteen wholes.

T: Should I write this a little bit so you can see it a little bit better? So I can take any whole number and write it as as fraction by just putting it over one? Sixteen oneths?

Class: Yeah, yes.

T: Well, that's kinda cool.

Line 40, 9:27 S__(Dalton): Awesome.

T: [Laughter] That is so awesome. I agree. So tell me, is math a beautiful thing or what?

Chapter 5: Summarizing 3.3C

Approximate time 09:37 – 11:07 (Times from start of video)

[3.3C, 1/3 of 2-1/2]

Line 1, 9:37 T: Let's listen, okay, guys?

S1 [Violet]: Okay, my wholes are these, and then I have half of them which is the yellow. So I split all of them into thirds - all of the pieces into thirds, and then I colored in one third from that, a third from that, and a sixth from the whole piece of that. Um, and then I added one third and another third and I got two thirds. I changed them over to four sixths so that I could add one sixth to that, and I got five sixths.

Line 10, 10:13 S__ [Unidentified]: Where'd the one sixth come from?

S1(Violet): My one sixth, that's my one sixth, and that was from that.

T: Did you follow what she did?

Class: Yeah, yes.

T: Okay, okay. So can I ask you something? What is this right here showing? Anybody.

Class: One sixth of a half - one third of a half.

T: One third of a half. So, to get this, would she have to do one third of one half, this times this?

Line 20, 10:45 Class: Yes.

T: Okay, that's how she got this little piece. And what is she showing here when she got two thirds?

Class: One third -

T: One third of two of them?

Class: Yeah.

T: So, one third of two - so, is it fair to say that she took a third of these guys and then a third of that half, and then she added and put them all together.

Class: Yeah.

Line 30, 11:05 T: How many of you did it similar to what she did?

Chapter 6: Summarizing 3.3D, Two Different Strategies

Approximate time 11:07 – 14:50 (Times from start of video)

[3.3 D, 10-1/2 x 2-1/3]

Line 1, 11:07 T: So how about if we start here? That's Violet again. We're keeping her busy today.

S1 (Violet): Alright. I did two thirds times ten halves, and then I changed two thirds to seven thirds, and ten and a half to twenty one seconds, and then three times two is six and seven times twenty one is a hundred forty seven, and I showed that here. So I did six, which is my denominator, divided by one hundred forty-seven, which is my numerator, and then I got twenty-four and three sixths.

Line 10, 11:50 S5 [Ellie]: So what was your answer again?

S1(Violet): Twenty-four and a half.

S__ [unidentified student]: You lost me with the three sixths.

T: Okay. So now somebody tell me what she just said. What's her strategy? Only two hands up? So, so the rest of you, what are your questions for Violet?

S__ [Unidentified]: She just converted the wholes into a fraction.

T: Hmmm. Can you do that?

Class: Yup.

Line 20, 12:23 S__ [unidentified student]: There's no rule that says you can't.

T: [Laughter]. So that's all she did? Nikki, what did she do?

S2(Nikki): She converted the wholes into a fraction.

T: So she took this, two and a third, and she made it seven thirds.

S__ [unidentified student]: She didn't like the idea of mixed numbers.

T: She didn't like the idea of mixed numbers, so she

Line 30, 12:41 switched them over so now she has two fractions. And then did she just use our algorithm, multiply the numerators and multiply denominators?

Class: Um huh.

T: Who did this one right here? Kristen, what did you do? Okay, watch up here guys, okay?

S11 [Kristen]: I knew I had to do two and a third, ten and a half times, and I found that one half of two and one third was one and one sixth, so, um, I added all the wholes, which got me twenty-one, and then I added all the

Line 40, 13:12 thirds, and three thirds is one whole and I had, um, one whole, two wholes, and then I had three wholes and a sixth and a third and one sixth and, um, I can't add those so I changed the third to two sixths and I added those and it was three sixths so altogether I got twenty-four and three sixths.

T: Was that the same thing Violet got?

Class: Yup.

T: So Kristen thought about it as repeated addition, didn't she?

Class: Yup.

Line 50, 13:47 T: What, what was our number sentence? How many of these two and a thirds do we have?

Class: Ten and a half.

T: We have ten and one half groups of two and one third. I heard her say something. How many of these does she have, two and one thirds, you guys?

Class: Ten.

T: Ten of them? So is it fair to say that the first thing she did was ten groups of two and a third, she just added them? Ten groups of two and a third, she just added them

Line 60, 14:11 all up as she went. So, what is this thing over here?

Class: That's half.

T: Half of what?

Class: Of two thirds, of two and one third.

T: Oh. So then she took a half of two and one third?

Class: Yeah, 'cause --

T: Can I do that? Would that, would that take care of it? Can I do ten groups of two and a third and then a half of a group of two and a third?

Class: Yeah.

Line 70, 14:40 T: So maybe I can leave them in mixed numbers. Yes? No? What --

S9 (Becca): Maybe 'cause that’s what helped me figure it out that, after I was having my big problem.

Chapter 7: Summarizing 3.3D, A Common Error

Approximate time 14:50 – 18:37 (Times from start of video)

Slide (showing student work for 3.3D)

Teri uses a drawing for a third correct strategy for part D to make a point about the Distributive Property.

Line 1, 15:00 S23 (Brooklyn): What I did is I drew two and one third for the acres for each day and then I added up two days so these two days together would be four and two thirds and I did that for each two days --

T: Could, could hers be a picture of Kristen's?

Class: Yeah.

T: Kristen used just numbers, and she used a picture. Now, I want to, I want to talk to you about something that I saw lots of you do. And I want you to think about, I

Line 10, 15:28 want to use that picture to think about why it doesn't work. So look up here for a second, okay? If I have ten and one half groups of two and one third, I saw some people trying to do ten times two is twenty, right? And then a half times a third equals one sixth, putting them together and getting twenty and one sixth. Okay? Why can we not just do that? Why does that not work? That doesn't give us enough. Why not? Allie, why not?

S8 [Allie]: Because you have to multiply both numbers. Like, you'd have to multiply ten times, um, a third and ten

Line 20, 16:10 times two and then a half, um, a half by a third and a half by a two.

T: Oh. So, do I have, I, can you guys see okay? Do I have ten groups of two and ten groups of a third in this picture?

Class: Yeah.

T: Do you see how I have ten two's but what we're forgetting about here is that we also have ten one-thirds in all of these days that she drew. Right? So I hear Allie saying even something different. I hear Allie

Line 30, 16:40 saying, let's do ten times two and ten times a third. And then a half of two? I don't know. Do I have a half, do I have two groups of a half somewhere?

Class: [Response unintelligible]

T: On this half day, on this half day, what do I have?

Class: One group --

T: So one, is that a half of two?

Class: Yes.

T: And then do I also have to do a half of that third?

Class: Yup, yes.

Line 40, 17:13 S (Becca): Mrs. _____, it's still the same as ten and two thirds and then –

T: You're absolutely right. What - go ahead.

S16 (Jesse): It's the same way when you're multiplying whole numbers, like fifty-six times twenty-two, you can't just split, um, split the fifty-six into fifty and six and multiply them separate. You have to multiply them together. You can't split both of them.

T: Okay, now, this is what I see and tell me if you see this. I see a couple of different ways to think about

Line 50, 17:47 this. I see this way, where we take any mixed number or any whole number and write it as a fraction, as an improper fraction, and then just use our algorithm. Hold on a minute. And then I see another way. I see kind of picking it apart and saying ten and a half twos, ten and a half of thirds, or really picking it apart. I see this sort of as one way, and this as another way. What are you thinking about those two strategies at this point? What are you thinking about them? Is there one that you're like, whoa, I like that one, I'm going to use that one. Or are you

Line 60, 18:25 kind of like, hmm, I still want to think about it. Tell me where you're at.

S__ (Jesse): I think the first one is more efficient.

T: This one?

S__ (unidentified): Yeah.

T: More efficient.

S9 (Becca): I like the ten and a half times --

T: You like this one.

Chapter 8: Teacher's Reflection 10/27/06
Approximate time 18:37 – 21:03 (Times from start of video)

Line 1, 18:37 T: Yes. The summary was absolutely beyond my wildest dreams today. [Laughter] I knew what I wanted to come out but I thought it was going to be, it was even better than I hoped. I knew that a couple of the examples that they had done, I was going to be able to use their example to get at this, take the whole number and put it over one. I thought I was going to have to tell them that. But that came right from one of the students. I thought I would be able to get at the distributive property, and that came from one of the

Line 10, 19:06 students. I thought I was going to have to tell them about, well, we could change mixed numbers to improper fractions, and lo and behold, when I was walking around before the summary, somebody was very quietly switching, I don't really like to work with mixed numbers, so I'm switching them over to improper fractions, and then I'm just going to use our algorithm. Aah, could you show that. That would be interesting. So, it was beyond my wildest expectations. I thought I was going to have to pull out a lot more than what got out today, and it's so much better

Line 20, 19:38 coming from them.••

T: Where have we been and where are we? Um, I think that, I think that we, my goal was for them to start making sense of multiplying with fractions, fractions and whole numbers, mixed numbers, mixed numbers. At least start thinking about all that, especially this part of a part business, 'cause that's so hard for them. I feel like we got there. I think it's still kind of fragile. I could tell by their homework that some of them were using the algorithm and trying to make their drawing fit the algorithm. So, that

Line 30, 20:07 concerns me a little bit. But, I think a lot of them at least are starting to think about what it means to find a part of a part and that we're getting this little piece, and I'm excited about that. I think my evidence is I hear them talking and I see it in their pictures. Um, I also think they're starting to think about what to do when they have a mixed number or a whole number. I don't think we're there. I, I'll, I'll probably, I'll come back on Monday. There's one more problem where they have to actually switch them all over to improper fractions and multiply. So we'll

Line 40, 20:40 probably come back to that. But I'll probably give them the option to use either way, 'cause I think some of them are more, felt more comfortable with distributing it out, and I don't, I think that's great. So, I'll probably go to the next problem on Monday, but give them the option. Do you want to switch them over? Go ahead and use Violet's way. If you want to distribute it out, go ahead and do it that way. And then we'll get to, to getting, to fixing our algorithm to fit all situations.