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Transcript for October 23- 27, 2006

"Skills Students Bring to *Bits and Pieces II*,
Investigation 3."

Students are shown using previously acquired skills,
as they develop new skills in
Bits and Pieces II, Using Fraction Operations,
Investigation 3:
Problem 3.1, "How Much of a Pan Have we Sold", and
Problem 3.2, "Finding a Part of a Part," and
Problem 3.3, "Modeling More Multiplication Situations."

The video was shot in real time and edited to 14 minutes.

Bits & Pieces II, Investigation 3
Class: 6th Grade
Date: October 23-27, 2006

Chapter 1: Whole Number Multiplication
Approximate time 00 – 00:53 (Times from start of video)

Slide:
How do students use whole number multiplication?
Problem: Students are discussing whole number multiplication. They have been asked for an estimate for 12 x 13.

Line 1, 00:20 Teacher: What are you expecting about this answer, twelve times thirteen? Are you expecting it to be big –

Class: Yeah.

Teacher: Or not? Yeah. Does anybody know how big?

Class: About a hundred.

Teacher: About a hundred.

Class: One fifty.

Teacher: About a hundred and fifty?

Line 10, 00:34 Class: Yeah,

S__ [off camera]: 'Cause twelve times twelve is one forty four.

Teacher: Okay. So if twelve times twelve is one forty four –

S__ [Hayden]: Well, plus twelve --

Teacher: What's thirteen, okay so what would that be?

Class: One fifty six.

Teacher: So the answer is, whoopsy, I'm sorry,
Line 20, 00:51 exactly one fifty six.

Class: Yes.

Teacher: Okay. Look how big it got.

Chapter 2: Whole Number Division
Approximate time 00:52 – 03:05 (Times from start of video)

Slide:
What do students understand about whole number division?

Problem: A whole pan of brownies costs $12.
a. You buy 2/3 of a ½ of a pan. How much does that cost?
b. You buy ¾ of a ½ of a pan. How much does that cost?

Line 1, 1:04 Teacher: How'd you get four dollars for that one?
S__ [Caleb]: That's two sixths of the pan --

Teacher: Right.

S__ [Caleb]: And two sixths of twelve dollars is, how did I do that?

Teacher: So if you got four dollars, how did you get four dollars?

S__ [Caleb]: I don't remember.

Teacher: What's that?

Line 10, 1:26 S__ [Caleb]: I don't really remember how I got that.
••

Teacher: Hmm. So what could we do here? You're saying three --

S__ [Caleb]: Oh, wait, yeah. Now I remember.

Teacher: Um, huh.

S__ [Caleb]: 'Cause if you divide twelve by six --

Teacher: I'm just going to write down what I hear you saying --

S__ [Caleb]: And then you took --

Teacher: You took twelve and divided by six -

Line 20, 1:41 S__ [Caleb]: And then that's two.

Teacher: So two, what does this mean, two what?

S__ [Caleb]: It means that each piece is worth two dollars -

Teacher: Oh. This is worth -

S__ [Caleb]: Two pieces

Teacher: Two dollars and this is worth two dollars?

S__ [Caleb]: Yeah. So then -

Teacher: Okay. Could I use that same idea here?

S__ [Caleb]: Not really, 'cause -

Line 30, 1:56

Teacher: Why?

S__ [Caleb]: You can't divide twelve by eight.

Teacher: Why not?

S__ [Caleb]: You'd get a negative.

Teacher: Do you? Why can't I divide twelve by eight?

S__ [Caleb]: Because it just won't go into twelve.

Teacher: Hmm. It doesn't?

S__ [Caleb]: [Unintelligible] one and a half.

Line 40, 2:18

Teacher: Is that, why couldn't we use that? Is it exactly one and a half?

S__ [Caleb]: Yeah.

Teacher: So what does this tell me?

S__ [Caleb]: That each piece is worth a dollar and a half.

Teacher: So if this is a dollar and a half, and this is a dollar and a half, and this is a dollar and a half, could I get my total cost?

S__ [Caleb]: Sweet.

Teacher: How did you decide this was one and a half?

Line 50, 2:49

How did you do that?

S__ [Caleb]: 'Cause eighths, a half of eight is four, eight and four would be twelve.

Teacher: Okay. So I can do twelve divided by eight.

S__ [Caleb]: Yeah.

Teacher: I just might not get a nice even dollar amount.

Chapter 3 : Meaning of the Denominator
Approximate time 03:05 – 03:58 (Times from start of video)

Slide
What do students understand about the meaning of the denominator?

Problem: 2/7 of 1/3

Line 1, 3:16 Teacher: They split every one of these three pieces, they split into seven pieces. So, how many pieces do I have in my whole thing now?

Class: Twenty one.

Teacher: Twenty one. Now, in this one third, how many pieces did they take?

Class: Two.

Teacher: Two. Hmm. So, two sevenths of one third. Look how little it got.

•Line 10, 3:36 Class. Um, hum.

Teacher: You know what. I'm noticing that every time. I'm noticing, that, ooh, I got a little two sixths, or a third. I got three eighths, I got two twenty oneths. My answer got so little. Why? Why am I getting a little answer? Yeah?

S_(Becca) Because the denominator is getting larger and that makes the pieces smaller?

Chapter 4: Subtraction of Fractions
Approximate time 03:58 – 07:19 (Times from start of video)

Slide:
What do students understand about subtracting fractions?
Problem: 17 ¾ - 4 16/20 (A student-generated "practice" problem in a warm up)

Line 1, 4:09 Teacher: So we're using our algorithms, right, that we made, that you made. OK. Listen up please.

Jesse: ……..Minus 4 equals thirteen. And then thirteen minus sixteen twentieths equals twelve and four twentieths.

Hayden: Why did you do 13?

Student(unidentified): He's not done yet.

Jesse: Well 13 because you minus….well I got that from there because I'm taking off the 4 and then I'm
Line 10, 4:56 taking off the fraction. And then I add back this fraction that's right here.

Hayden: But how do you know 13, because sixteen twentieths…..

Nikki: Because he took the seventeen and he took the fourteen, the wholes and he minussed them. Then he had thirteen and he had… didn't have enough …..

Jesse: three fourths is equivalent to fifteen twentieths. And then you add those….twelve and nineteen twentieths.

Line 20, 5:41 Teacher: I think Jesse uses a different algorithm than the algorithm that we have in our notes.

Jesse: What I do is… I just take … so I minus the wholes first so I have 13, and then I minus the fraction. But I forget…well … I just like leave this fraction out of it and minus the fraction from the wholes, but then I got to add this back, cause that's still part of this.

(Another student comes to the board)
Drew: Well first I times three fourths by five over
Line 30, 6:22 five so it'll be easier to subtract and that got me 15/20. So then there will be seven, no seventeen and 15/20 minus 4 and 16/20 and you can't subtract 16 from 15, so you cross this out and make it a 16. This would be 35 minus 16 and that equals 19.

Teacher: Thirty five twentieths you got altogether?

Drew: No. Cause I crossed that out and added the whole so that

Teacher: Right

Drew:……..so that equals nineteen twentieths, and then
Line 40, 7:19 sixteen minus four equals twelve.

Chapter 5: Meaning of Fractions
Approximate time 07:19 – 09:40 (Times from start of video)

Slide:
What do students understand about the meaning of fractions?
Problem: 5/6 x ¾

(In the background we hear the teacher talking to other students, but the focus is on the actions of the students on camera.)

Line 1, 7:30 Teacher: Hey Guys! We're going to spend one more minute on this. So whatever you're going to put up on the board can't take more than a minute. OK?

(Class noise in the background)

Teacher: can you do it in a minute? If you can do it in a minute put it up there.

Student (unidentified): Is this different?

Teacher: I don't know because I don't know what you did.
……(unintelligible) ….. cut it into what, four pieces?
Line 10,7:53 And then what did you do when you took a fourth?
(unintelligible) To me it looks like Skyler's cause you cut it into six pieces and took five….. Looks a lot like what Skyler did.)

Jesse: Or…

Dalton: I'll circle them.

Teacher: Do you guys see how he got five sixths of this fourth, five sixths of this fourth and five sixths of that fourth? And then kind of put them all together. Is that all right? And then some other
Line 20, 8:52 guys did that same problem a little bit differently.

Dalton: well first I cut it into, umm, fourths. There's a fourth there, a fourth there and a fourth there. And then I just cut each fourth into half, because if you cut each fourth into half, there would be six, cause I cut it….

Jesse: Cause there's only three fourths.

Dalton: Yeah, that'd be six, and then I colored in five, and I counted these two and there'd be eight, so I got five eighths. (unintelligible)

Line 30, 9:31 Jess: And then I just found the equivalent.

Teacher: OK. So same amount, just two different ways to look at it. Fifteen little teeny pieces or five bigger pieces.

Chapter 6: Commutative Property of Multiplication
Approximate time 09:40 – 14:01 (Times from start of video)

Slide:
What do students understand about the commutative property for multiplication?
Problem: Is 2/3 of ¾ the same as ¾ of 2/3?

Line 1, 9:51 Teacher: Does everybody have their ACE out? And their homework out? Now remember if it's something that somebody had a question on and talk about and you need to rethink while we are talking and you want to make some revisions, go ahead before I collect it. OK? If you came without your homework today you don't get this privilege. If you don't have your homework you know where you need to go right? OK. So, 4A. Is it A? 4A and B. It says "Use a Brownie
Line 20, 10:20 Pan model to show whether finding 2/3 of ¾ of a pan means the same thing as ¾ of 2/3 of a pan."

Nikki: First I just drew a box. And then I split it up into… fourths.

Teacher: I'm really sorry. Are you doing 2/3 of 3/4 right now or the other one?

Nikki: I'm doing 2/3 of ¾.

Teacher: OK.

Nikki: And I colored in three of them for three
Line 30, 10:57 fourths. And then I cut it in thirds. And I… then I colored two of them.

Teacher: (Laughs). It's not there. Is it? OK.

Nikki: And then I got twelve sixths, six twelfths.
Teacher: Could you write that number sentence for us? What did she just draw, you guys? Tell her what she just drew.

Everyone: 2/3 of ¾.

Teacher: OK. And what did you end up with?

Nikki: Six twelfths or a half.

Line 40, 11:37 Teacher: OK. All right.

Nikki: And then I just knew you could do it the opposite way because you just start out with thirds instead of starting out with the fourths.

Teacher: Could you just try it over there? Seeing is believing, right? So, now what is she going to start with, guys?

Class: Thirds.

Teacher: Thirds. If we take ¾ of the 2/3.

Nikki: Thirds. Colored in two of them.

Line 50, 12:03 Teacher: Feel free to move if you can't see something, OK. Go ahead and move if you're trying to see and you can't. That's fine. There's her two thirds

Nikki: And then I ….. and then I colored in three of the fourths and then I got the same answer. One two three four five six, seven

Student(unidentified): It's just the same as what you….

Nikki: Yeah, you just rotate it. You can just rotate the box.

Line 60, 12:45 Teacher: Aww, you're kidding. Can you just rotate that?

Class: Yeah.

Teacher: So how does that piece over there that you double colored compare to this piece over here that we double colored?

Students(unidentified): Because that part…. Same answer… just flip flop…….

Hayden: The part that she doesn't have right there is on the bottom ……

(Teacher: talking to a student who had to leave, while Nikki finishes writing on the board.)

Line 70, 13:21 Teacher: So what do you guys think, those of you who had a question on number 4? What do you think? OK? Did you get your question answered?

Student: (unintelligible)

Teacher: For 4B. I thought we talked about that. "If the brownie pans are the same size, how do the final amounts compare?" So you've got 6/12 there and 6/12 there.

Student(unidentified): Both be the same.

Teacher: Yeah. Same amount.

Line 80, 13:48 Hayden: I've got a question. It says at the end …it just… (unintelligible)

Teacher: OK. Then you can make a revision if you want. So what does that say about 2/3 time ¾ and ¾ time 2/3.

Student(unidentified): It's commutative.

Teacher: Commutative. Exactly.

Chapter 7: Estimation
Approximate time 14:01 – 17:23 (Times from start of video)

Slide:
How do students use estimating skills?
Problem: Estimate ½ of 2 9/10, 1 ½ x 2, 2 ½ x 4/7

Line 1, 14:12 Teacher: We're just going to estimate how much this might be before we try to figure out some sort of a strategy. Could you help me with an estimate? If I said I was going to get one half of two and nine tenths, Drew, how could we think about estimating that answer?

Drew: Well, nine tenths, that's close to a whole, so -

S (unidentified): _____ one.

Drew: Yeah, so -

Line 10, 14:39 Teacher: So what, Drew?

Drew: Nine tenths is close to a whole.

Teacher: Okay.

Drew: So you could round that up into a whole and then two would be a three.

Teacher: So you would call this about three?

Drew: Yeah.

Teacher: And then can you think about what it would be to be half of three?

Drew: Um -

Line 20, 14:58 Teacher: What's half of three?

Drew: One and a half.

Teacher: What if I want one and a half groups of two and nine tenths. So it's not a part of a whole, but it's part, groups of, we're sort of back to groups again. What if I have one and a half groups of this. Drew, could you pick somebody to help us estimate this one.

Drew: Um, Hayden.

Hayden: Um, I think, I think that the wholes would
Line 30, 15:14 add up so the one and the two, I'd put -

Teacher: We're adding them up?
Hayden: Well I think you'd have to times it 'cause one times any number is always itself, so I think it'd be about four and a half because -

Nicky: Oooh.

Hayden: We just, because -

Teacher: Because why?

•Hayden: Because if I was going to round up the nine tenths like Drew did so -

•Line 40, 15:36 Teacher: So you're going to call this three? Okay.

Hayden: Three times one equals three and then the half, er Teacher: So -

Hayden: It'd be -

Teacher: You're doing three times one is three and then what are you doing with your half?

Hayden: I was going to just add that.

Teacher: Just add it on and call it a half?

Hayden: Three and a half, not four and a half.

Teacher: Oh, okay.

Line 50, 15:54 Hayden: I did it wrong.

Teacher: What do you guys think about that?

S_(unidentified): I think it's _____

Teacher: Brooklynn, does that make sense?

Brooklynn: I don't think it would be three and a half because if you did one times three it would be three but if you did a half times three then it would be one and a half.

S_(unidentified): _____

Teacher: Okay. So you're saying you would think
Line 60, 16:12 about it like one times three, which would give you three, and then you're taking a half of three also?

Brooklynn: No, half three times.

Teacher: Oh, a half three times.

Brooklynn: Yes.

Teacher: Like this?

Brooklynn: Um hmm.

S_(unidentified): Ooooh.

Teacher: And that would be -

Hayden: That's how I got four and a half.

Line 70, 16:30 Teacher: Oh, and then you would take this and this and put it together?

Hayden: I couldn't figure out -

Teacher: Oh.

Hayden: How I got three and a half - four and a half at first.

Teacher: Oh, so now we're going to come back and four and a half makes sense?

Hayden: I couldn't remember how I got that -

Teacher: That's okay.

Line 80, 16:40 Teacher: How are we going to think about - how are you going to think about estimating something like - just estimating.

S_(unidentified): I think it _____

Teacher: Brett, can you see okay? How, how can we think about estimating that one? Kristen, you have any ideas about how to estimate that one?

Kristen: Well, um, I think it might be, um, well, 'cause if it was four and - four sevenths, um, times two and one half it would be like four sevenths of
Line 90, 17:03 two and a half.

Teacher: Okay, so you're thinking about it like this?

Kristen: Yeah. So -

Teacher: Okay.

Kirsten: And four sevenths is about half so I think it'd be like two.

Teacher: So you're thinking of half of two and a half.

Kristen: Yeah.

Teacher: So what would a half of two and a half be?

Kristen: Oh, wait. It would be, um, one and one fourth.

Chapter 8: Rewriting Fractions
Approximate time 17:23 – 18:32 (Times from start of video)

Slide:
What do students understand about rewriting fractions?
Problem: 2/3 of 16.

(classroom noise – teacher's voice in the background)

Line 1, 17:40 Kaylie: (counting pieces) Thirty two sixteenths.

Ellie: No, threes cause you cut each pan into three.

Kaylie: Oh.

Ellie: Thirty two threes

Kaylie: (unitelligible)

Ellie: No, no (erasing)

Kaylie and Ellie: Thirty two threes

Ellie: So then there's … (unintelligible) …32.

Line 10, 18:11 Ellie: See I got 10… because we know, three goes into thirty 10 times-

Kaylie: Why do we have to do that-

Kaylie: Why'd we have to do that-

Ellie: Because we have to make it into… so it's a proper fraction -improper?

Kaylie: Oh, okay.

Ellie: So if we make this a thirty, then it would go in ten times, and then two over three would be your proper fraction.

Kaylie 10 and 2/3.

Line 20, 18:28 Ellie: See, and that's what I got by doing this. I got 10 and 2/3.

Chapter 9: Relationship between Multiplying and Dividing
Approximate time 18:32 – 21:29 (Times from start of video)

Slide
How does dividing whole numbers relate to multiplying fractions?
Problem: 2/3 of 16.

Line 1, 18:43 Caleb: I did circles, and circles count for one ounce, 'cause the problem was chocolate chips, ounces. And I, so I made sixteen of those, and then I grouped them up, I grouped them up into fives, 'cause that's the closest I could find to sixteen being divided by an even number. And then, the last one I cut into thirds and added to each group, so I came up with a third of sixteen being five and a third, and then I - (Off-camera comments from class:
Line 10, 19:16 I don't…)

Nikki: You lost me when you said…You just started talking really fast and then you lost me.

Teacher: Okay.

Nicky: Like after you put -

Teacher: Okay. So do you all understand that he has sixteen circles there to show sixteen ounces?

Class: Yeah.

Teacher: Okay. Now, can you back up and go from there, Caleb?

Caleb: Alright. I took the sixteen ounces and I
Line 20, 19:36 grouped them off into fives -

Teacher: And how, why were you doing that?

Caleb: 'Cause that's the closest to an even number that I could get to, well, a whole number, that I could get to go into, uh, sixteen three times.

Class: Ooh. One third…

Teacher: Okay.

Caleb: The closest I could get -

Teacher: Brett - excuse me just for a second. I'm just going to try to write something. Can you come
Line 30, 19:55 over here for a second so I can get by your paper? [*Teacher goes to board]* Caleb, is it true that you're trying to take sixteen and split it into three groups?

Caleb: Yeah.

Teacher: Okay. Now you following what he's doing?

Class: Yeah.

Teacher: Why would Caleb trying to be split it into three groups? Why? Because what?

Class: To find two thirds -

Line 40, 20:12 Teacher: Yeah. Because you're trying to get thirds, okay? And he said five's the closest he could get, isn't it?

Class: Yeah.

Teacher: Okay. So go from there, Caleb.
[Camera returns to Caleb at board]

Caleb: Then I got the one that was left over from when I grouped it off into fives. Then I cut that into thirds, and just put a third with each group of fives.

Line 50, 20:31 Class: Oooh.

Teacher: You following?

Class: Yeah.

Caleb: So that made it so a third of sixteen was five and a third, and then I just multiplied that by two and got ten and two thirds.

Ellie: Why did you multiply it by two, if you have –

Caleb: Because five and a third is one third, and I need two thirds.

Ellie: Oh.
(Class makes positive comments.)

Line 60, 20:55 Teacher: how is doing this [16 ÷ 3] like this [1/3 of 16]? Are those two the same thing or different things?

Class: Same.

Teacher: Are they the same thing?

Class: Yup.

Teacher: Say more about that. They're the same?

Jesse: They're the same. Well, they're the same because, well, you're dividing that sixteen, you're dividing it into three pieces, which aren't three
Line 70, 21:21 pieces, thirds, so you're dividing that into thirds, and the second one, when you're saying one third of sixteen.

Chapter 10: Prior Skills with Fraction Operations.
Approximate time 21:29 – 24:55 (Times from start of video)

Slide
What skills do students use in developing a strategy for multiplying mixed numbers?
Problem: 1/3 of 2 ½

Line 1, 21:39 Violet: Okay. My wholes are these, and then I have half of it, which is the yellow. So I split all of them into thirds, all of the pieces into thirds, and then I colored in one third from that, a third from that, and the sixth from the whole piece of that. Um, and then I added one third and another third and I got two thirds. I changed them over to four sixths so that I can add one sixth to that, and I got five sixths.

Line 10, 22:10 Class comments: Um hmmm. Yeah. _____ one sixth, 'cause -

Violet: My one sixth, that's my one sixth, and that was from that [pointing to third bar in drawing].

Slide:
What skills do students use in developing a strategy for multiplying mixed numbers?
Problem: 10 ½ x 2 1/3

(Another student comes to the board for a different problem)

Kristen: Um, I knew I had to do two and a third ten and a half times, and I found that one half of two and one third was one and one sixth, so, um, I added all the wholes, which got me twenty-one, and then I added all the thirds, and three thirds is one whole, and I had one whole, two wholes, and then I had three
Line 20, 2:51 wholes and a sixth, and a, one, a third and one sixth, and um, I can't add those so I changed the third to two sixths and I added those and it was three sixths, so altogether I got twenty-four and three sixths.

(A third student comes to the board for the same problem as Kristen solved.)

Brooklynn: What I did, is I drew two and one third for the acres for each day, and then I added up two days, so these two days together would be four and two thirds, and I did that for each two days. And then I added them up and that would make them nine
Line 30, 23:25 and one third. And then I knew that I had a sixth over here, so I changed it to nine and two thirds *[sic]* [pointing to nine and two sixths], and then, um, I had, let's see, I had day nine and ten, which were the extra ones I had, 'cause I only went to day eight, and I added that with the half a day, and I switched the two thirds to four sixths and that got me five and five sixths and then I added these two [9 2/6] and this [5 5/6] - first I added these two [9 2/6] together and that got me eighteen and four

Line 40, 24:06 sixths, and then I added this answer [5 5/6] to - with the eighteen and four sixths, and that got me twenty-four and three sixths.

(Violet comes back to the board to do the same problem as Brooklyn and Kristen, another way.)

Violet: OK. I changed two thirds [pointing to 2 1/3] to seven thirds and ten and a half to twenty-one seconds, and then three times two is six and then seven times twenty-one is a hundred forty-seven and I showed that here. So I did six, which is my denominator, divided by *[sic]* one hundred forty-seven, which is my numerator, and then I got twenty-four and three sixths.

[Class comments].

Line 50, 24:53 Ellie: What was your answer again?

Violet: Um, twenty-four and a half.

Chapter 11: Understanding the Distributive Property
Approximate time 24:55 – 26:30 (Times from start of video)

Slide:
How does multiplying whole numbers relate to multiplying mixed numbers?

How are these examples of the Distributive Property?
Problem: Why is 10 ½ x 2 1/3 not just 20 1/6?

Line 1, 25:06 Teacher: If I have ten and one half groups of two and one third - I saw some people trying to do ten times two is twenty, right? And then a half times a third equals one sixth - putting them together and getting twenty and one sixth. Okay. Why can we not just do that? Why does that not work? That doesn't give us enough. Why? Why not? Allie, why not?

Allie: 'Cause you have to multiply both numbers. Like you have to multiply ten times, um, a third and
Line 10, 25:43 ten times two, and then a half, um, a half by a third and a half by a two.

Jesse: It's the same way when you're multiplying whole numbers. Like, fifty-six times twenty-two. You can't just split, um, split the fifty - fifty-six into fifty and six and multiply them separate. You have to multiply them together. You can't split both of them. You can split one but not both, like –

[Jesse runs to board]

Jesse: You'd have to do this, like twenty-two times fifty and then

Slide:
Showing what you can and can't do when multiplying 56 x 22.