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"Making Connections: *Moving Straight Ahead*, 2.3 and 2.4"
(16 chapters, approx 35 minutes)

LAUNCH: Before viewing the video "Making Connections"

Do the Problems	**Before participants view "Making Connections" they need to do Problems 2.3 and 2.4.** I will probably not take the time to do 2.3 and 2.4 in the same way as I would with students, but I will make posters with some "teacher questions" for participants to consider as they do these Problems. The five "teacher questions" below have worked well in the past. They help teachers go beyond just doing the Problems, and they prepare teachers to watch students do the same Problems.

Getting Ready to view the video	**Possible "Teacher" Discussion Questions:**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
	• How might students recognize that a relationship (for example, between cost and number, in 2.3) is linear?	- *They should recognize linearity from a table (constant rate), from a graph (a line), and from an equation.* - *They should see the given equations in 2.3 are linear.* - *They might have trouble seeing that the table in 2.3B represents a linear relationship.*	- *What is it about the table in 2.3B that hides the "linearity?" How can we help students think about this, without telling them?*[Ω]

[Ω] We might ask questions like, *Can you tell how much 9 T-shirts would cost? How did you work that out? Why would 0 T-shirts cost $34? Can you tell how much a single T-shirt costs from this table? Does it always cost the same?* The constant rate per *extra* T-shirt is obscured by both the $34 deposit, and the fact that the number of T's is not increasing by constant increments in this table. We can't say whether the rate per additional T-shirt is a constant without taking into consideration how the independent variable is changing. This encourages a more careful definition about what we mean when we say that the "Y is increasing at a constant rate."

Cont'd	**Possible "Teacher" Discussion Questions:**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
	• How does recognizing linearity enhance student strategies for completing tables (as in 2.3B) or in solving problems (as in 2.3A)?	• *Once you have established that a relationship is linear you can extend a table using the constant rate idea.* • *If you know two equations represent linear relationships then you can draw lines and find the point where the lines meet.*	- *Can you find the equation that generates the table in 2.3B? How might this help solve 2.3B?* - *If two relationships were not linear would finding an intersection point of their graphs be possible? Helpful?*
	• How does finding a point on a graph of a line relate to solving a linear equation (as in 2.4 C and D)? How many linear equations can be solved from the graph of one line?	• *Is that what 2.4 C and D are about? I didn't really get the question.* • *(2, 4) is on the graph of Plan 2* ***because*** *(2, 4) is a solution for Plan 2's equation. – and vice versa. You can think of a graph as a picture of all the solutions for an equation.* • *(2, 4) is a solution for Plan 2's equation, so are (3, 3) and (4, 2) and (5, 1) etc. $y = -x + 6$ has an infinite number of solution pairs, each matching a point on the graph of the line.*	- *Why can't we look for a solution for $8 = 5x - 3$ on any of the other graphs of Plans?*

Cont'd	**Possible "Teacher" Discussion Questions:**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
	• What are some difficulties you might expect students to have with these Problems?	• *Students may think Plan 3 pledges $2 per kilometer.* • *The "-x" in Plan 2 may confuse students. It looks like we are subtracting kilometers (x) from dollars (6). The missing coefficient, -1, may throw them. This relates to the C-Mighty equation also.*	- *Would a table help students see the missing coefficients?*
	• Are graphical and tabular ways of solving equations, as in 2.4D, as valuable/ more valuable than symbolic ways?	• *Solving 8 = 5x -3 is faster and more accurate by using symbolic ways.*	- *Could we solve –x + 6 = 5x – 3 symbolically? What would it mean in this context?*[Ω] - *Could we solve $x^2 = 5x - 3$ symbolically? $x^3 = 5x - 3$?* [Ω]

[Ω] Teachers often have a preference for solving linear equations symbolically, perhaps because of their own mathematics education. I want them to connect each such solution with a point on a particular line, (2.2, 8) on y = 5x – 3, in the case of 2.4D, or (2,4) on y = -x + 6, in the case of 2.4C. There are an infinite number of points on y = 5x – 3, each of which is related to an equation b = 5x – 3. However, there is only one point on both lines y = 5x – 3 and y = -x + 6. We can solve –x + 6 = 5x – 3 symbolically or graphically.

[Ω] The first of these can be solved using the quadratic formula or a graph or table. There are 2 solutions. The second can be solved using a graph or a table. There are 3 solutions. The table below may help teachers see the value in graphical or tabular methods.

X	$Y_1 = x^3$	$Y_2 = 5x - 3$
-3	-27	-18
-2	-8	-13
-1	-1	-8
0	0	-3
1	1	2
2	8	7

Can you tell from the table: Which graph will be lower? How does this help you locate approximate solutions? How can you make the solutions more accurate?

VIDEO: "Distraction or Making Connections"
(*Moving Straight Ahead*, 2.3 and 2.4, 15 chapters, 31 mins. + Reflections)

Note: This video has been edited to focus on students making mathematical connections. Real time is 2.5 class periods.

EXPLORE: While watching the video

This video has been cut down from 2.5 class periods, about 130 minutes, to just over 30 minutes. We only see 25% of what transpired on these days. Students have just made their own definitions for *y-intercept* and *coefficient of x* when the video begins, and they bring with them the mathematical knowledge from Investigation 1, and Investigation 2, Problems 2.1 and 2.2, plus knowledge from prior units. From the video we can try to ascertain: How do they connect this knowledge?

Focus Questions to consider while watching the video

My focus questions are all about analyzing connections students make, with or without the help of the teacher. It would work best if different participant groups focused on different phases (for example, a group might focus just on the Launches or Summaries) or on different Problems (2.3 or 2.4). We can jigsaw their observations in the large group summary.

Student groups seen on video:

- **Kristen, Bryce Kelsey**
- **Kelsey, Jocelyn Melanie Lily**
- **Aaron, Ricky Taylor Logan**
- **Travis Becca Jayna**
- **Tyler Sean Emmett**

- What connections did the teacher or students raise in the Launch phase for 2.3?
- What evidence is there that students are making connections during the Explore phase of 2.3?
- What was the teacher's role during the Explore phase of 2.3?
- What connections did the teacher or students raise in the Summary phase of 2.3? What was the teacher's role?
- What connections did the teacher or students raise in the Launch phase for 2.4?
- What evidence is there that students are making connections during the Explore phase for 2.4?
- What connections did the teacher or students raise in the Summary phase of 2.4? What was the teacher's role?
- What Connections did students miss? What confusions still need attention?
- How important is it that students make connections?

It has worked well in the past to allow time for participant groups to clarify the questions *before* viewing the video. In small group discussions *after* viewing the video I have an opportunity to ask follow up questions, before orchestrating a large group discussion.

	Focus Questions (as above):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
SUMMARIZE: Discussion in small and large group after watching the video	• What connections did the teacher or students raise in the Launch phase for 2.3?	• *The teacher asked if the situation was linear. Students focused on the constant rate in the context, the cost per T-shirt.* • *One boy said it was like the Mugwumps.*	• *What connection is the boy seeing to the Mugwump activity in Comparing and Scaling?* • *What are the pros and cons of delving further into the Mugwump connection at this time? (The teacher did investigate this connection a little more than we see on the video.)*
	• What evidence is there that students are making connections during the Explore phase of 2.3?	• *Students talk about y-intercept as the "start," and the coefficient as the "rate."* • *One boy connects the y-intercept to a personal situation, the deposit for a trip.* • *One student connects n to n•1 and then says they are the same. He's looking at the symbols.*	• *Were students connecting graphs, tables and equations to each other and to finding solutions?*

Cont'd	**Focus Questions (as above):**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
	• What was the teacher's role during the Explore phase of 2.3?	• *She asks clarifying questions, especially when students use language in an unclear way.* • *She asks probing questions.*	• *Can you give examples of the questions she asked? Were they effective?*
	• What connections did the teacher or students raise in the Summary phase of 2.3? What was the teacher's role?	• *The coefficient of x in both equations was connected to adding the same for each T-shirt- which sounds like a connection to the table.* • *The teacher asked about how a graph might have helped.*	• *How did the students handle the coefficient of x in y = 49 + x? Is it worth making this 1x more explicit at this point? Where does the "1" appear on the table or graph?*
	• What connections did the teacher or students raise in the Launch phase for 2.4?	• *They connected a point on the graph to the answer to a question.*	• *Is it worth pushing for the connection to the solution to an **equation** at this point? (12 = 5 + 0.5(14)) Would this take anything away from the cognitive demand of 2.4D if it is discussed in the Launch? What does the Teacher Guide say about this?*

Cont'd	**Focus Questions (as above):**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
	• What evidence is there that students are making connections during the Explore phase for 2.4?	• *Becca might be making a connection to order of operations when she tries to understand the meaning of the expression 5x – 3.* • *Some students connected the table to the setup for the axes for the graphs.*	• *The teacher asked Lily's group to think about y-intercept and coefficient, or a table, as a way to solve their puzzle about the meaning of y = -x + 6. How might this connection help them?*[Ω] • *What has Aaron done with his graph that puzzles Ricky?*[Ω]

[Ω] If Lily's group is able to identify the coefficient of x as -1, and the y-intercept as 6, then they may be able to interpret this as "starting with $6 and going down $1 for every kilometer walked." If they are able to produce a table, either by hand or calculator, they would see the same thing. This might make the *-1* dollar/kilometer rate more explicit for them. At least one other group was not able to see that the *x* is multiplied by -1 in this equation.

[Ω] Aaron just graphs 2 points from the table and joins them. Ricky is concentrating on plotting each point in the table. It might be worth pointing out that Aaron's line picks up all of Ricky's points and more. Are these additional points also solutions for the equations?

Cont'd	**Focus Questions (as above):**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
	• What connections did the teacher or students raise in the Summary phase of 2.4? What was the teacher's role?	• *Students got the connections between the x-coefficient and the behavior of the graph.* • *The teacher writes the equation $y = -x + 6$ to accompany Travis's explanation for 2.4C. This makes the connection more obvious.*	• *One boy brings up that the point after (2, 4), on $y = -x + 6$, would be (3, 3). What connections does he seem to be using?* • *What do we make of Travis's method of solving $8 = 5x – 3$ symbolically? What are the pros and cons of following up on this in class?*

Cont'd	Focus Questions:	In Previous Workshops Teachers Have Said:	Follow Up Questions
	• What Connections did students miss? What confusions still need attention?	• *I don't think all students understood why we would look at the graph of $y = 5x – 3$ in order to solve $8 = 5x – 3$.*	• *What might we do to help students clarify which graph helps solve $8 = 5x – 3$?*[Ω]
Note: Save this question for the wrap-up discussion.	• How important is it that students make connections?		

[Ω] It might help to look at another equation, say 5 = 5x – 3, as a whole class, because, although everyone chose the correct graph, no one corrected Travis's reasoning. A large poster-size graph of all three plans would make the discussion easier to orchestrate. If we follow Travis's strategy of drawing a horizontal line, at y = 5 for this equation, we find that this line intersects 2 of the graphs drawn. *Which one should we choose? Suppose we choose the graph of $y = -x + 6$. What point do we find? ((1, 5)) Suppose we choose the graph of $y = 5x – 3$. What point do we find? ((1.6, 5)) Which of these points helps us answer $5 = 5x – 3$? (Substituting $x = 1$ in this equation does not make the equation true. Substituting $x = 1.6$ does make the equation true.) Where would we look to find the answer for $6 = 5x – 3$? $7 = 5x – 3$? $9 = 5x – 3$? How many equations can we solve by looking at the graph of $y = 5x – 3$? Can we choose any point on $y = 5x – 3$ and make up the equation/question that this point solves/answers? Suppose we want to solve $5 = -x + 6$. Now which graph should we look at?*

SUMMARY CONT’D
Culminating Discussion

After small groups have had an opportunity to talk about the above focus questions, with me listening in and asking follow up questions as necessary, we can have a cross-group discussion. I might have each small group make a poster of their comments on their focus question. And then I can re-assemble the groups and have everyone rotate through the posters, comparing and discussing findings.

As a large group we can then share common findings, points that we disagree on, and answer the last question. *Why is it important that students make connections? What advantage is there in knowing more than one way to solve an equation, and how those ways are connected? What more do we understand by being able to recognize linear relationships from different representations?*