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"Distraction or Learning Opportunity: Reprise of *Moving Straight Ahead*, 2.1"

LAUNCH: Before viewing the video "Distraction or Learning Opportunity"

Note:
Student work for 2.1 is needed for this introductory discussion.

Note:
It may be sufficient to for participants read Problem 2.1 and look at the student work - not actually take time to view the video of 2.1.

Before participants view "Distraction or Learning Opportunity" they need to understand how students solved Problem 2.1, and why students thought the Problem was unrealistic. This may mean viewing and discussing "Students Using Representations of Linear Relationships." The video "Distraction or Learning Opportunity" does not stand alone.

I think I will launch into the study of "Distraction or Learning Opportunity" by briefly reprising the teacher discussion about the learning trajectory in *Moving Straight Ahead*, as in the launch for "Students Using Representations of Linear Relationships". With student work from 2.1 in hand participants can try to assess where students are in relation to Investigation 2 and Problem 2.1 goals. However, we also have to address the question implicit in the title.

- Which of the main goals on page 2 (Teacher's Guide) are the focus of Investigation 1? Which are not dealt with in Investigation 1?
- How do the goals of Investigation 2 advance the mathematics in Investigation 1? How do the goals of Investigation 3 connect to the goals of Investigation 2?
- How do the goals for Problem 2.2 build on the goal for Problem 2.1?
- How do we know whether we should follow a student question or comment? Can we predict whether it will distract us from our mathematical goal or enrich student understanding?

The point of this discussion is to instill the idea that teachers need to know the overall Unit, Investigation and Problem goals so they can take advantage of student work in the exploration phase to orchestrate effective summaries. The development of understanding of the mathematical goals is their guide to what questions to ask, and whether to follow up on student questions and ideas.[Ω]

[Ω] It is not possible to know exactly what will come out of mathematical side trips suggested by students. Experienced teachers try to predict if a side trip will further mathematical goals. Sometimes the teacher does not exactly understand what a student is asking about. Sometimes the teacher knows that the mathematics behind a student question is dealt with later. Sometimes the teacher sees an opportunity for enrichment or differentiation. Balancing opportunities against time available requires a deep knowledge of the curriculum. We see one example playing out on this video.

Small group discussions of the preceding questions should suffice, if participants have already done Problem 2.1 and/or viewed the video "Students Using Representations of Linear Relationships."

After the brief introduction above, I have, in the past, used discussion questions to help participants prepare their mindsets to watch the video.

Getting Ready to view the video	**Possible "Teacher" Discussion Questions:**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
Note: Participants should have access to student work for 2.1 throughout this discussion.	• What strategies did students use to solve 2.1? (See student work.)	• *Students created and analyzed tables and graphs.* • *Some students have combined their knowledge of the equations with some efficient "guessing and checking" strategies.* • *Some students just "guess and check" randomly.*	- *No group set up an equation to answer the question, "What race time would make the two distances equal?" What would this equation look like?* [Ω] - *The difference in distances walked is 45 meters. The difference in rates is 1.5 meters/ second. How are these related?* [Ω]

[Ω] 45 + t = 2.5t. It is probably too soon to expect students to independently think of this. Solving this kind of equation is a focus of Investigation 3.

[Ω] The distance between the boys starts as 45 meters and decreases. For most students the distance between the boys at the end of the race was 1.5 meters. This shrinking distance appears on the graph and the table. The difference between the walking rates is 1.5 meters/second. How long will it take for the difference in rates to wear away the head start? 45 = 1.5t?

Possible "Teacher" Discussion Questions cont'd:	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
• How are these strategies related to the goals for Investigation 1? To the goals for Problem 2.1? 2.2?	• *Recognizing, creating, working with, and understanding linear tables, graphs, and equations was an main goal of Inv 1.* • *2.2 focuses on using different representations to solve problems, and connecting different representations of solutions.*	
• Several students thought the large head start made it obvious that Emile was letting Henri win. What might be gained by allowing students time to create more "realistic" problems?	• *A deepening understanding of connections among rates of change, slope, y-intercept and intersection points can take place, since students are confident with context from their first effort with 2.1.*	- *What connections are there among rates of change, y-intercept and intersection points? How do these relate to students trying to make the Problem more real?*[Ω]
• Do you expect that students will use the same strategies when they solve their more "realistic" problems as they did in Problem 2.1?	• *Many kids will continue using their original strategies as they create new but similar situations.*	- *Would you "push" kids to solve their new problems with different strategies than they used with 2.1? What are the pros and cons of doing this?*

[Ω] The walking rates dictate the slopes of the graphs. The greater the difference in slopes (rates) the faster Emile catches up, so the greater the head start (y-intercept) must be. If we want to reduce the headstart we have to make the line representing Emile's (time, distance) relationship have a slope that's only a little steeper than Henri's. The intersection point represents a time and distance when Emile would catch up to Henri. Any time-distance pair (on Henri's graph or table) less than this would be a solution.

VIDEO: "Distraction or Opportunity"
(*Moving Straight Ahead*, 2.1, 12 chapters, 28 mins)

Note: This video has been edited to focus on student strategies for creating and solving an alternative to Problem 2.1. Real time is 1.5 class periods or 75 minutes.

EXPLORE: While watching the video

This video has been edited to focus on 5 groups of students creating and solving their own problems, in which 2 brothers race, but the older brother arranges the race so that the little brother barely wins. Many students were quite concerned that the head start was over half the total race distance in Problem 2.1. They all tried to "improve" the problem by making the walking rates closer and the head start shorter, and by choosing a combination of time, distance or head start that would let the little brother win.

Focus Questions to consider while watching the video

My focus questions should be all about analyzing what students understand or do not understand, assessing their mathematical development in terms of the goals of Investigation 2, and discussing the costs and benefits of permitting students to follow up on their interest in creating an "improved" problem. (I think I will not show the last chapter, the teacher's reflections, until after participants have had an opportunity for both small and large group discussions.)

Student groups seen on video:

- **Kristen, Bryce Kelsey**
- **Kelsey, Jocelyn Melanie Lily**
- **Aaron, Ricky Taylor Logan**
- **Travis Becca Jayna**
- **Tyler Sean Emmett**

- Do all groups find a solution? Do the created problems and their solutions seem more realistic than the original problem?
- What are some strategies that different groups used?
- Does each group repeat their thinking from the previous day? Does any group change or extend their thinking?
- Can we compare strategies from Day 1 to Day 2? Does anything mathematically or pedagogically useful arise from this kind of comparison?
- Since groups are no longer solving exactly the same problem can we make comparisons among Day 2 strategies? Do we need to do this to meet the goals of 2.1 and 2.2?
- Does a linear equation like $y = mx + b$ appear in any solutions? Does any group actually solve an equation?
- What are some advantages of allowing students to devise their own problems as replacements for 2.1? Disadvantages?
- Are there any mathematical goals for 2.2 still outstanding after the summary phase? What, if anything might we do about this?

It has worked well in the past to allow time for participant groups to clarify the questions before viewing the video. In small group discussions after viewing the video I have an opportunity to ask follow up questions, before orchestrating a large group discussion.

	Focus Questions (as above):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
SUMMARIZE: Discussion in small and large group after watching the video	• Do all groups find a solution? Does their problem and its solution seem more realistic than the original problem?	• *Some groups have benefited from seeing others' solutions for the original Problem 2.1. Equation-generated tables and graphical solutions are now part of each group's conversation.* • *Every group has found a solution with a shorter head start.*	
	• What are some strategies that different groups used?	• *All groups seem to have decided that the key is to alter the rates to allow for a shorter head start and a more realistic race.* • *The same strategies appeared again (guess and check, tables, graphs, substitution in equations) but graphs appeared or were mentioned more often.*	- *Are the students focusing on finding a particular solution, or on a strategy? Does it bother them that everyone is solving a different problem now?* - *Is there an advantage in having calculator table and graph strategies more explicitly mentioned?*[Ω]

[Ω] Because the problems are all different the focus is more on the idea that the intersection point (whether on table or graph) is key, rather than on what that particular intersection point is. This foreshadows setting up and solving an equation, or a system of equations.

Focus Questions (as above):

- Does each group repeat their thinking from the previous day? Does any group change or extend their thinking?

In Previous Workshops Teachers Have Said:

- *Jayna, Becca and Travis again guess and check different headstarts and race distances to find close times.*
- *Kelsey's group uses a calculator table generated by equations on Day 2, instead of substituting in isolated equations.*
- *Bryce and Kristen spontaneously use the graphing calculator on Day 2.*
- *Tyler's and Ricky's groups seem to be using the same strategies as on Day 1.*

Follow Up Questions:

- *Becca notes that the head start should relate consistently to the time difference by which Henri wins, but their calculations do not show this. How might we help clear up the confusion we see on the video?*[Ω] *Is this worth investigating with the whole class?*
- *Does the use of the graphing calculator extend students' thinking?*

[Ω] Becca is correct in expecting a pattern; however, her statement is not completely logical. Assuming all the head starts chosen allow Henri to win, then the *greater* the head start the *greater* the time difference between he and his brother when Henri completes the chosen race distance. Assuming walking rates of 2 and 2.5 meters per second, they have calculated these results:

Head Start	Time Difference
5	2(Travis)
6	1 (Jayna)
7	1.5 (Becca)

This group may not be ready to write equations, $d = 5 + 2t$, $d = 6 + 2t$, $d = 7 + 2t$, and $d = 2.5t$. *If* they can relate their proposed solutions to these equations then a graph (or table), showing how distances, times, and time differences relate to each other, might help this group. How does time *difference* appear on the graph of $d = 2.5t$ and $d = 5 + 2t$, for example?

Jayna and Becca discover later that Henri is actually *losing* with the above head starts. It also turns out that Jayna has been assuming a 40 meter race, Becca has been assuming a 50 meter race, and Travis has been assuming a 45 meter race. We do not find this out until the teacher asks all students to re-group and share their solutions with other students. When this is discovered Becca's group decide that they should be using the same race distance and they re-work their solution successfully.

Focus Questions (as above):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
• Can we compare strategies from Day 1 to Day 2? Does anything mathematically or pedagogically useful arise from this kind of comparison?	• *Several groups seem to be more systematically looking for the intersection point.*	• *How does Kelsey's group's Day 1 strategy relate to her Day 2 strategy?*[Ω]

[Ω] On Day 1 Kelsey's group uses 2 equations, d = 2.5s and d = 1s + 45, to guess at and substitute different values for s, and compare the resulting values for d. They write :

Emile: d = 2.5s	2.5(15) = 37.5	2.5(18) = 45	2.5(29) = 72.5
Henri: d = 1s + 45	15 + 45 = 60	18 + 45 = 63	29 + 45 = 74

On Day 2 Kelsey's group uses a graphing calculator to make a table. Notice they are creating a new problem, with different rates and head start.

X	$Y_1 = 2.5x$	$Y_2 = 30 + 1.5x$
0	0	30
1	2.5	31.5
…		
28	70	72
29	72.5	73.5
30	75	75

The calculator is now doing the substitution and calculation and the students are focusing on finding equality between the distances.
How is searching the table like solving 2.5x = 30 + 1.5x?

We might write their solution as (29, 73.5).
How we can check this solution in the equations.
Note: it solves y = 30 + 1.5x not y = 2.5x.

Focus Questions (as above):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
• Since groups are no longer solving the same problem can we make comparisons among strategies or solutions? Do we need to do this to meet the goals of 2.1 and 2.2?	• *Three groups have actually used the same parameters, a head start of 10 meters and rates of 2 and 2.5 m/sec. Their solutions are (15, 40), (19, 48) and (19.5, 49). We could compare all these solutions on the same graph.*	• *How are the solutions of these three groups similar?* • *Why does Becca question the +10 in Tyler’s equation?*[Ω] • *How do these 3 solutions relate to the graphs? How many solutions are possible with rates 2 and 2.5, and head start 10?*

[Ω] Tyler’s equation for the race distance is 10 + 2t = d , or 10 + 2(19.5) = 49. To Becca this seems as if the little brother is being given 10 *extra* meters to walk. Becca’s group starts by assuming a race distance of 40 meters and *subtracting* a 10 meter head start. Becca seems to think that “d” in Tyler’s representation is the distance walked by *each* boy; Tyler’s “d” is the “40” in Becca’s representation. A direct comparison of these strategies, and of the underlying equations, might help the class see the similarities.

Becca	**Tyler**	**Bryce**	
40 = 2.5t 40 ÷ 2.5 = 16 seconds	d = 2.5t d = 2.5(19.5) d = 48.75 meters	d = 2.5t d = 2.5(19) d = 47.5 meters	What does Becca’s and group start by assuming? What does Tyler’s group start by assuming? How is *40 = 2.5t* like *40÷2.5 = 16*?
40 – 10 = 30 30 ÷ 2 = 15 seconds	d = 10 + 2t d = 10 + 2(19.5) d = 49 meters	d = 10 + 2t d = 10 + 2(19) d = 48 meters	What do *2t* and *10 + 2t* represent? How does the solution (15, 40) relate to Tyler’s equation? Can we start with d =49 meters and use Becca’s guess and check method?
Solution (15, 40)	Solution (19.5, 49)	Solution (19, 48)	

Becca’s strategy assumes a race distance (and rates) and solves for times. Tyler assumes a race time (and rates) and solves for distances. They both start with the underlying relationship: d = rt (or d = rt + b), though Becca’s group thinks of d ÷ r = t (or d – b = rt).

Focus Questions (as above):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
• Does a linear equation like $y = mx + b$ appear in any solutions? Does any group actually solve an equation?	• *Several groups use equations like $d = 2x + 10$, but usually with the intention of making a graph or table.*	• *How does what Jayna writes relate to solving $40 = 10 + 2x$?*[Ω] • *How does the equation $2t + 10 = 2.5t$ relate to what Tyler and Bryce are doing?*
• What are some advantages of allowing students to devise their own problems as replacements for 2.1? Disadvantages?	• *Students are motivated by ownership in what makes a good problem.* • *The summary can focus less on particular solutions than similarities among strategies.*	• *How are graph, table and equation strategies connected?*
• Are there any mathematical goals for 2.2 still outstanding after the summary phase? What, if anything might we do about this?		

[Ω] Jayna, Becca and Travis have devised as strategy which seems to foreshadow solving linear equations like 40 = 10 + 2x. They do not seem to see the underlying equation at this point. We might ask about the simpler equation, 40 ÷ 2.5 = 16. How do they know to divide the race distance by 2.5? Is *distance* ÷ *rate* = *time* always true? How is this related to *distance* = *rate x time*? How does 40 – 10 = 30 relate to 40 = 10 + 2t? Why is their next step 30 ÷ 2 = 15? Jayna, Becca and Travis seem to have developed an "undoing" strategy for solving a = b + cx for x.

SUMMARY CONT’D Culminating Discussion

After small groups have had an opportunity to talk about the above focus questions, with me listening in and trying to keep the focus on the mathematical ideas which we see students using on the video, we should have a large group discussion about the costs and benefits of making time to follow student interest. The last two focus questions relate to this.

The teacher’s goal is to be able to use student ideas to enable the whole class to reach the goals of Problem 2.1 and 2.2. Participants must judge whether these goals have been satisfactorily dealt with after Day 2, and whether they would have been just as satisfactorily dealt with after Day 1. If participants judge students to be further ahead on Day 2 than they were on Day 1, then we can confidently say there were mathematical gains. If participants judge that Day 2 has not added anything mathematically useful to student knowledge, then we still have to assess what else might have been gained by honoring students’ questions about the practicality of the original Problem 2.1.

I think that mathematical gains were made; most groups extended their thinking to take into account the ideas they saw briefly on other groups’ posters in Day 1. And I think that the classroom environment of respect for others’ ideas and commitment to making sense are the result of similar teacher decisions. But I should not push my ideas on participants. That would be contrary to the spirit we see on the video.

The last chapter of the video is the teacher reflecting on her students’ progress, her goals for them, and her instructional decisions. It would be fitting to allow the teacher the last word.