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"Students Using Representations of Linear Relationships: *Moving Straight Ahead*, 2.1"

LAUNCH: Before viewing the video of students doing *Moving Straight Ahead*, 2.1.

Before viewing the video participants need to do *Moving Straight Ahead*, 2.1. This is the first step in planning to teach any lesson.

I think I will "launch" 2.1 by having a teacher discussion about the learning trajectory in *Moving Straight Ahead*. We can refer to the Teacher's Guide, giving new teachers an opportunity to use this resource.

Note:
There are several good articles that address the issue of orchestrating good summary discussions. (See References.) I may assign "Orchestrating Productive Discussions" (Smith et al) to be read prior to the workshop.

- *Which of the main goals on page 2 (Teacher's Guide) are the focus of Investigation 1? Which are not dealt with in Investigation 1?*
- *How do the goals of Investigation 2 advance the mathematics in Investigation 1?*
- *Which of the main goals on page 2 is a focus of Investigation 3? How do the goals of Investigation 3 connect to the goals of Investigation 2?*
- *How do the goals for Problem 2.2 build on the goal for Problem 2.1?*

The point of this discussion is to instill the idea that teachers need to know the overall Unit, Investigation and Problem goals so they can take advantage of student work in the exploration phase to orchestrate effective summaries. The development of the mathematical goals is the guide to what questions to ask, whether to follow up on "what if" questions, and how far to push students. We need to know how the next Problem (or Investigation) builds on the current Problem (or Investigation) so we know what we *must* push for and what can wait.

Then we will do Inv 2.1 in its entirety, Launch- Explore - Summarize, just as if the participants are students. In the Summary phase I should be sure to ask explicitly for connections and comparisons among strategies.[Ω] Small group discussions of 2.2 should suffice.

[Ω] In this Problem many participants will arrive at a solution of 29 seconds, 74 meters. I should ask how this appears in the table, on the graph and in the equations. Specifically, does (29, 74) appear on *each* line, and in *each* table? Does it fit *both* equations? Is there one point (or solution) that lies on *both* lines/tables/equations? If someone proposes a different solution (any time less than 30 seconds and corresponding distance for Henri is a solution) we should try that in both equations, both graphs and both tables. Comparing the location of different solutions on the tables and graphs helps people see that many solutions are possible, and how the graph, table and equation show solutions.

After the "student" summary of 2.1, I have, in the past, used "teacher" questions, which help participants prepare their mindsets to watch the video of students doing the same problem.

After participants have done 2.1: Getting Ready to view the video	**Possible "Teacher" Discussion Questions:**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
	• What strategies do you expect your students will use in Problem 2.1?	• *Students will make tables and easily find a solution.* • *Kids might guess and check.* • *This problem is too difficult because it is too open.*	• *Should we push for multiple methods and how could this impact student understanding?*
	• What difficulties might you predict for your students in 2.1?	• *Students will have difficulty dealing with a 45 meter head start in this situation.* • *Students will have difficulty with the lack of a 1 in the coefficient in Henri's walking rate.*	• *Is it important to talk about the coefficients at this stage?*
	• How do the goals of 2.2 advance the goals of 2.1?	• *The goals for 2.2 include finding and* ***connecting*** *graph/table/equation* ***solutions****.*	

Possible "Teacher" Discussion Questions cont'd:	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
• Were the goals of 2.2 addressed in "our" summary? If "yes," how was this done? If "no," what remains to be done?	• *Yes, they were. We compared different groups' strategies and found that a solution on a graph could also be substituted in an equation, or found on a table.* • *No. I know that we compared strategies but I think each group should try each strategy, so I think that 2.2 is still necessary.*	• *What does 2.2 as written in the student text add to students' experiences?*
• What will your role be in the Summary?	• *I will orchestrate the summary*	• *What do you mean "orchestrate?" What specifically will you do to plan for the summary?*

VIDEO: "Students Using Representations of Linear Relationships" (*Moving Straight Ahead*, 2.1, 14 chapters, 21 mins.)

Note: This video has been edited to focus on student strategies for solving Problem 2.1. Real time is 1class period or 55 minutes.

EXPLORE: While watching the video

In this video we see 5 groups of students thinking about *Moving Straight Ahead* Problem 2.1. Each group finds a solution to the problem, by using a table or a graph or by substituting in equations.

Student groups seen on video:

- **Kristen, Bryce and Kelsey**
- **Kelsey, Jocelyn, Melanie and Lily**
- **Aaron, Ricky, Taylor and Logan**
- **Travis, Becca, Jayna**
- **Tyler, Sean, Emmett**

In *Moving Straight Ahead* Investigation 2 the focus is on comparing linear relationships, *using* representations of two linear relationships to solve problems, and *connecting the solutions* found by using different representational strategies. The summary of 2.1 has to go beyond sharing different strategies; comparing, connecting and analyzing are called for. However, this is not the only opportunity students have to make connections among solution strategies and representations.

With this in mind I think I will ask each group of participants to follow a **particular group** of students closely, so they can identify with these students and explain the thinking of these students to the rest of us. Transcripts and student work should also be available, so we can use actual students' words as evidence. After we view the video I will ask participants to role-play the students, and I will ask one small group to plan for and take on the role of the teacher in a summary phase that should continue on from where the video ends. If the participants are new to CMP then I might have to help them plan for the summary.

I need to propose some focus questions, to guide the viewing. These focus questions should be all about what the students understand, where the students are in relation to the goals of this Investigation, and what the teacher can do to add mathematical value to the summary phase of the lesson. After we have acted out the summary we should discuss whether we think students will still need to do Problem 2.2. Obviously how the summary plays out will affect our decision, so the discussion is a very realistic enactment of the kinds of instructional decisions teachers have to make.

To introduce the video I should remind teachers of what they said they expected from their students and what difficulties they anticipated. *These predictions as the first step in planning a lesson.*

Focus Questions for Teachers and Teacher Leaders

Each participant group should view the entire video but should apply the following focus questions to their particular student group.

- What strategy did your group of students choose? Is this strategy effective? Is this strategy similar to any strategy chosen by us (participants in workshop)?
- How does the mathematics used by your group of students appear to relate to their mathematical experiences in Investigation 1? To the mathematical goals for Problem 2.1? To the mathematical goals for Problem 2.2?
- What is the teacher's role when the students are exploring? Are her actions and questions effective in bringing out the mathematics? In developing classroom norms for investigation? What are the students doing during the "explore" phase? Are their actions and conversations effective in addressing the mathematics?
- What would you like to ask your student group during the explore phase?
- How does the strategy or representation of your group of students connect to the strategies or representations of other groups you see on the video?
- What would you like to ask your group of students during the summary phase?

Post Focus Questions.

Handouts with Focus Questions.

It has worked well in the past to allow participants time to talk about the focus questions *before* they view the video as well as time to debrief in small groups *after* the video. Having the focus questions posted in the front of the room, as well as on individual handouts with room for notes, keeps participants focused.

I have noticed a tendency for people to make general comments in response to the video. Follow up questions make the discussion richer.

	Focus Questions (as above):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
SUMMARIZE: Small Group Discussion after Viewing the Video **Note: Make student work available**	• What strategy did your group of students choose? Is this strategy similar to any strategy chosen by us (participants)?	• *Our student group chose to substitute in the equations. We did this too, but we used the calculator (table) to do this.* • *Our student group guessed and checked without mentioning equations. No teacher group did this.* • *Our student group made a table. The teacher asked them to make a graph. Most of our teacher groups made graphs.*	- *The group that guessed and checked without equations was Travis's group. How is their method like solving $70 = 45 + t$ and $70 = 2.5t$? (see also the note on page 9.)*
.	• How does the mathematics used by your group of students appear to relate to their mathematical experiences in Investigation 1? To the mathematical goals for Problem 2.1? To the mathematical goals for Problem 2.2?	• *Every group except Travis's seemed to be able to make tables and relate these to equations. Some were able to use graphs. (Goals for 2.1) I don't see that they connected a **solution** found on a graph to a solution found on a table or in an equation. (Goals for 2.2)*	- *What might we ask these students in the Summary to connect graph/table/ equation **solution**s?*

Focus Questions (as above) cont'd:

- What is the teacher's role when the students are exploring? Are her actions and questions effective in bringing out the mathematics? In setting the classroom norms? What are the students doing during the "explore" phase? Are their actions and conversations effective in addressing the mathematics?

- What would you like to ask your student group during the explore phase?

In Previous Workshops Teachers Have Said:

- *The teacher seems to not "tell" anything. She pushes the kids to clarify their thinking.*
- *The kids seem to be very engaged. How did the teacher create this climate of engagement?*
- *The teacher tries to have students share and explain ideas, rather than working individually.*
- *I would like to ask Ricky's group how the walking rates appear in their tables, and how the numbers in the equations relate to their tables.*
- *I would like to ask Kelsey's group what 2.5t represents and what 45 + t represents. Do we want to know when these are equal?*
- *I would like to ask Kristen's group how the rates and head start appear on their graph/ table.*

Follow Up Questions:

- *What actions is the teacher taking to create this climate of learning?*
- *How has the choice, sequence, and depth of tasks to this point in the unit impacted the "engagement" of kids learning/working?*

Focus Questions (as above) cont'd:

- How does the strategy of your group of students connect to the strategies of other groups you see on the video?

In Previous Workshops Teachers Have Said:

- *Travis's group's solution doesn't connect to other groups.*
- *Both Travis's and Tyler's group seem to be guessing and checking, but Tyler's group is guessing times and calculating distances, while Travis's group is guessing distances and calculating times.*
- *Tyler's group seems to be using equations like Kelsey's group, but they did not write general equations down.*
- *Ricky's and Bryce's groups both made tables, but they did not use the same increments.*

Follow Up Questions:

- *How does the common solution of (29, 74) appear on the graph and table, and how does Travis's solution of $d = 70$ and $t = 25$ compare to this? Can we use tables and equations to compare these two different solutions?*[Ω]

[Ω] Travis's strategy was to start with a guess for the race distance and calculate the corresponding times. He refined his guess to make the times closer. Groups who used a table or equations seem to be starting with the guesses (or substitutions) for times and calculating the corresponding distances; they all explicitly or otherwise mention the relationships, $d = 2.5t$ and $d = 45 + t$. It's not clear that Travis's group realize how their calculations of $70 – 45 = 25$, and $70 \div 2.5 = 28$, relate to these two linear relationships. Asking them how they knew to subtract 45, and to divide by 2.5 might reveal that they also are thinking implicitly of these equations. In fact what Travis has done is solve $70 = 2.5t$ and $70 = 45 + t$. All groups might benefit from seeing this. It would be interesting to compare the points (29, 74) and (25, 70) on the graph. Notice these are points on Henri's line only. This raises the issue of there being multiple solutions, (26, 71), (27, 72), (29.5, 74.5) for example. The challenge is how to ask questions to draw attention to these connections without just telling.

Focus Questions (as above) cont'd:

- What would you like to ask your group of students during the summary phase?

In Previous Workshops Teachers Have Said:

- *Tyler, how does your solution appear in Bryce's table or graph?*
- *Bryce, how does your table relate to Ricky's table?*
- *Ricky, would Travis's solution appear in your table if you had not skipped lines? Where?*
- *Kelsey, do your guesses appear in either of the table solutions?*
- *Travis, could we substitute your solution (25, 70) in the equations Kelsey wrote?*

Follow Up Questions:

- *No group set up an equation to answer the question, "What race time would make the two distances equal?" What would this equation look like?*[Ω]
- *Tyler raises the issue of the distance between the boys. How does that appear on the graph? How does the distance between the boys relate to their rates?*[Ω]

[Ω] $45 + t = 2.5t$. It is probably too soon to expect students to independently think of this. Solving this kind of equation is a focus of Investigation 3.

[Ω] The distance between the boys starts as 45 meters and decreases. For most students the distance between the boys at the end of the race was 1.5 meters. This shrinking distance appears on the graph and the table. The difference between the walking rates is 1.5 meters/second. How long will it take for the difference in rates to wear away the head start? $45 = 1.5t$?

SUMMARY CONT'D

When the video is over each participant group needs time to debrief and study the student work. This gives me an opportunity to ask follow up questions as needed (see above).

Large group discussion: Plan for and Role play the Summary, and Evaluate Student Progress

Each group should then post on newsprint their answers for the last three questions (*What questions would you like to ask your group during the Explore phase? During the Summary phase? How does the strategy of your group connect to the strategies of other groups?*), so we can discuss as a large group the mathematics the summary might draw out.

Note: Sometimes participants are very wary of taking the teacher's role. I may have to be satisfied with a discussion here.

We should then **role-play the summary** of this lesson. If the participants are new teachers then I will coach a group as they plan to take the role of the teacher. We should be sure to draw on participants' ideas from the preceding discussion. I might ask the group leading the summary to think about how to *sequence* the student work and what questions they might ask to draw attentions to *connections*. (Smith et al. See Appendix)

Large Group Summary cont'd

Other questions we might ask during the summary are: *How does your strategy relate to the ideas developed in Investigation 1 or foreshadow the goals of Investigation3?*[Ω] *What connections do we see among different strategies*.[Ω]

We have to address the question of whether the goals for 2.1 and 2.2 have been met. *Will students have to do 2.2 to meet all the mathematical goals?*[Ω] *Or can we add to the Summary to address all goals of 2.1 and 2.2?*

I would like to end by reminding all the participants of the five practices in the article we read, and reflecting on the planning process. *Did we try to predict student strategies? Did we monitor students' thinking during the explore phase, with a view to the mathematical goals of the problem? Did we select student work and sequence the student work for the summary phase, with a particular purpose in mind? Did we connect, compare and analyze different strategies?*

[Ω] In Investigation 1 students were investigating patterns in representations of linear relationships. The patterns in the tables in 2.1 reflect the rates of the boys and also the way that the distance between them shrinks. We hear students using these patterns to shortcut their work in creating the tables, and getting to the common point. In Problem 2.1 students are in fact searching for the intersection point of the two lines: $y = 2.5x$ and $y = x + 45$. Any value of x less than the x-coordinate of this point will be a solution to how long in seconds the race must be. The corresponding value of y (for Henri) will be a solution to how long in meters the race should be. This solution will appear in the table (for Henri) also. It is unlikely that students will set up the equation $2.5x = x + 45$, but equations do come into play in the solutions shown on the video. Solving equations formally occurs in Investigation 3.

[Ω] Since different groups produced tables, graphs and equations for the *same* underlying relationships we can ask about connections. For example, where does the intersection point on the graph appear in the table? What does it mean? Where does Travis's group's solution (a 70 meter race) appear on the graph? (There will be a point for Henri (25, 70) and a point for Emile (28, 70).) Where does Travis's group's solution appear in the table? How does Travis's group's solution compare to the Guess-and - Check –in - the - equations strategy used by Tyler's group? What other solutions could there be? How is each group's solution like a coordinate pair for Henri (and a coordinate pair for Emile)? How are the table and graph solutions, (29, 74), related to the equations written by Jocelyn's group? How is Travis's group's solution related to these equations? How does the distance, or time difference, between the boys appear in the table or graph? Students have other opportunities to address these connections in 2.2, 2.3 and 2.4.

[Ω] The decision about whether 2.2 is necessary will depend on how the summary of 2.1 goes. Kathy, the teacher on the video addresses this issue in her reflections on Day 2.