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"Making Sense of the Concept and Representations of Exponential Decay: *Growing, Growing* 4.2"

LAUNCH:
Before viewing the video of students doing *Growing, Growing*, 4.2.

Do the Problem

Before viewing the video participants need to do *Growing, Growing 4.2*. The mathematics in this problem might be unfamiliar to some teachers, so I probably have to take the time to do a Launch – Explore – Summary as if the participants were students. I focused on the teacher's role in the L-E-S process for the video of students doing *Growing, Growing* 4.1, so this time I will focus just on the mathematics, and on evidence of student reasoning about the mathematics.

After the "student" summary of 4.1, I can use "teacher" questions, which help participants prepare their mindsets to watch the video of students doing the same problem.

Getting Ready to View the Video: After participants have done *Growing, Growing* 4.2	**Possible "Teacher" Discussion Questions:**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
	• How is this problem different from 4.1?	- *Students can't just divide by a whole number for 4.2 part B.*	- *How might they do 4.2B?*
	• What are all the mathematical ideas in this problem? Are they sequenced helpfully in the parts of the problem?	- *Part A gives a table and graph similar to the table and graph in 4.1, so they should be able to find the equation. Part B focuses on how the decay factor relates to what is lost or left.*	- *How does growth rate relate to growth factor? How does this compare to the relationship between decay rate and decay factor?*[Ω]

[Ω] Students already know that a growth rate of 20% causes a growth factor of 120%, or 100% + 20% additional each time. The connection between a decay rate of 20% and a decay factor of 80% is not so obvious, but it is the same idea; the decay factor is 100% - 20% subtracted each time. The idea that 20% that is lost and 80% remains should come out of part B.

Possible "Teacher" Discussion Questions (cont'd):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
• Will students have enough knowledge from 4.1 to get into 4.2?	- *They should be able to do part A. Part B will cause discussion.*	-
• What are likely difficulties for students? Do we want to address any of these in the Launch, or is it better to let them struggle?	- *Students are likely to confuse decay factor and decay rate when they write the equation.*	- *Would it have helped to talk in the Launch about the table given for part A? What does the 400 represent? What does the 100 represent? Why is the first decrease 300 and the next decrease only 75? How does this appear in the graph? Or would this discussion give too much away?*

Video: "Making Sense of Symbols: Exponential Decay" (*Growing, Growing,* 4.2)
9 chapters and 22 minutes

This video has been edited to focus on evidence of student understanding, and the teacher's role in assessing understanding and adding value to the Summary phase.

EXPLORE: While watching the video

The following focus questions are intended to help participants listen carefully to student discussions, with a view to assessing what mathematical ideas they understand, what they are still struggling with, and deciding what we, as teachers, might ask to focus the discussion effectively. The primary audience for these questions is teachers.

Each person should focus on one or two questions while watching the video.

Focus Questions

- Did students' words and actions fit your predictions? Were the difficulties as you predicted?
- Would you want to add anything to the Launch to help students with the idea they are struggling with?
- Give an example of a student interchange during the Explore or Summary phase that is interesting mathematically. What would you like to ask the students? What is the purpose of your question? (assessing understanding, scaffolding, reviewing, connecting, probing, re-voicing, summarizing etc.)
- Give an example of students' words that embody either the main idea in 4.2 or illustrate their confusion with the idea.
- Did students connect their solution for 4.2 to prior work with exponential decay or exponential growth? What might you ask to help make this connection?

Brief Focus Group Discussion

It has worked well in the past to re-arrange participants into focus groups before viewing the video. If they have a few minutes to talk about the focus question *before* the video and then time to debrief in small groups *after* the video I have noticed that the discussions are more coherent. I have tried to think of follow up questions that will help participants extend their thinking.

SUMMARIZE: Focus Group Discussion after watching the video.	**Focus Questions (as above)**	**In Previous Workshops Teachers have said**	**Follow Up Questions**
	• Did students' words and actions fit your predictions? Were the difficulties as you predicted?	- *They got confused about how the decay rate fit into the equation. Some of us had problems with the same idea.* - *I was interested to see students explain this to each other.*	-
Note: Alternative ways to conduct discussions: It can be unnecessarily repetitive if the same discuss/ view/discuss format is followed in every pd session. I have tried different formats. Some of these are described in the appendix.	• Was there anything in the Launch, or would you want to add anything to the Launch, to help students with the idea they are struggling with?	- *I think we should spend time in the Launch looking at the table for part A together, as we discussed after **we** did the problem.* - *I think we should let students struggle. We can analyze the table later if needed.*	- *What are the pros and cons of using the Launch phase to analyze the table in 4.2A? Where can we see the fraction of the original medication that is left and the fraction that has been used up?*
	• Give an example of a student interchange during the Explore or Summary phase that is interesting mathematically. What would you like to ask the students? What is the purpose of your question?	- *Logan and James disagreed about the equation. James says that 12 is 20% of 60, pointing at his table, where he has 12 mg after 1 hour. We might ask James what the 12 means.*	- *What is the intended effect of your question for James?* - *What might we ask the group that is subtracting 20% each time, to extend their thinking?*[Ω]

[Ω] We might point out to this group that they just defined decay factor as something that involved multiplying or dividing, not subtracting. Their strategy is correct, but can they accomplish the same effect without subtracting?

Focus Questions (cont'd)	**In Previous Workshops Teachers have said**	**Follow Up Questions**
• Give and example of students' words that embody either the main idea in 4.2 or illustrate their confusion with the idea.	- *James says that the table in part A shows the medicine "decreases 25% each time." He adds a clarification, "It loses a fourth." He then agrees with Logan that "you have one fourth what you originally had." This seems contradictory.*	- *The teacher revoiced what James and others said about the table in part A. What do you think her purpose was? Was she successful?*[Ω]
• Did students connect their solution for 4.2 to prior work with exponential decay or exponential growth? What might you ask to help make this connection?	- *They are asked in A3 to compare the graphs for 4.1 and 4.2A.*	- *Which problem, 4.1 or 4.2A, has the faster decay rate? How does the graph show this? What else can we ask about the two graphs?* - *If a quantity has a growth rate of 20% per hour, then the growth factor is 120%, or 1.2. What is the "1" in this factor? How does this relate to finding the decay factor when the decay rate is 20%?*

[Ω] Kathy is trying to have them think hard about what the table shows, in terms of what is lost and what remains. This is the language that successful groups had used for part B in the Exploration phase. I might want to ask the class if the two statements "decreases by 1 fourth each time" and "you have a fourth left each time" are equivalent.

FINAL SUMMARY: **Large group discussion after viewing the video**	After participants have had an opportunity to talk in their small groups we should have a large group discussion. This gives me an opportunity to ask Follow Up questions as needed. If this workshop is typical then some participants will have struggled with the mathematics in this problem, so I can expect that there will still be mathematical questions at this time.