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"Making Sense of the Concepts and Representations of Exponential Decay: *Growing, Growing*, 4.1"

LAUNCH:
Before viewing the video of students doing *Growing, Growing*, 4.1.

Note: I may assign "Orchestrating Discussions of challenging Tasks" as a reading prior to this workshop. (Smith et al. See Appendix.)

Before viewing the video participants need to do *Growing, Growing 4.1.* I think I will launch 4.1 by having a "teacher" discussion of the ideas about exponential *growth* students will bring to this investigation, and what they need to get from 4.1. Then we will do Inv 4.1 in its entirety, Launch- Explore and Summarize, just as if the participants are students.

After the "student" summary of 4.1, I can use "teacher" questions, which help participants prepare their mindsets to watch the video of students doing the same problem.

Getting Ready to View the Video:	**Possible "Teacher" Discussion Questions:**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
After participants have done 4.1 **Questions for Teachers**	• What equations do you predict your students might generate?	- *They will try to use 64 and 2 in some combination.* - *They are likely to try to use what they know about exponential growth, and build on* $y = a(b)^x$*, so they may try to alter* $y = 64(2)^x$. - *They may try dividing or subtracting because of the decrease. So,* $Y = 64 - 2^x$ *or* $y = 64 \div 2^x$. - *Some will use* $y = 64(1/2)^x$.	- *What questions might you ask to get your students started to think about an equation?*
	• What difficulties do you predict for your students?	- *They may not see the standard equation* $y = 64(1/2)^x$*. The context makes you think of dividing.*	- *Is it a problem if students use the "division" version of the equation? Why?*

Possible "Teacher" Discussion Questions (cont'd):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
• Is there anything that I might have included in the Launch that would have helped you (or students) be successful exploring 4.1?	- *Maybe we could have looked at the table of data for the original ballot problem, and talked about the pattern of growth.*	- *What should students know about the pattern of exponential growth?*
• What was I doing during the Explore phase?	- *Looking for different solutions* -	- *How might I decide to select student work for sharing in the Summary? Should I show all student work?*
• What will students understand just by doing the problem?	- *The shape of the graph* - *The pattern in the table* - *An equation* - *Comparison with exponential growth*	- *What connections can we see in the 3 representations? What might we ask students to draw attention to connections?*
• What were my reasons for specific choices of "student" work for the Summary phase?	-	- *Might I have sequenced the "student" work differently?* - *Did I draw out connections?*[Ω]
• What do students still have to understand about exponential decay after doing 4.1?	- *They need to understand that the graph and equation show what is left. Cutting in half each time makes the equation and pattern easy, compared to a decrease of, say,10% each time.*	- *What is the equation that shows a* ***growth*** *rate of 5%? 50%? 500%? How does this relate to the equation for a* ***decay*** *rate of 5%, 50%, or 500%?*

[Ω] See the article by Smith et al for a list of useful practices. See Appendix.

	Possible "Teacher" Discussion Questions:	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
Questions for Curriculum leaders	• Is Exponential growth or decay part of your curriculum? Part of your State standards?	- *Not at Middle School level. I am surprised to see it here.*	- *How does the pattern of growth or decay compare to linear relationships? Is the comparison of patterns helpful?*[Ω]
	• Do you think teachers are going to be familiar with this material?	- *I am quite concerned. Teachers who are already feeling unsure about their ability to orchestrate lessons are going to find it an extra challenge when they also feel unfamiliar with the mathematics.*	- *Has your school or district thought about helping teachers become confident with the mathematics of Growing, Growing or other algebra units?*

[Ω] Students have spent a great deal of time thinking about constant rates of increase or decrease, that is, linear relationships. The contrast between an additive pattern ("Add 2, add 2, etc") and a multiplicative pattern ("Multiply by 2, multiply by 2 etc.") brings out why the linear pattern of increase produces a graph that is a straight line with constant "steps," and why the exponential pattern of increase shows a curve with constantly increasing "steps."

VIDEO: "Making Sense of the Concept and Representations of Exponential Decay: *Growing, Growing*, 4.1"

31 minutes, 11 chapters.

The video has been edited down from a real time of about 1.5 hours to 31 minutes. We follow the learning trajectory from when students first see an example of exponential decay to where they feel confident enough to attempt a definition of decay factor.

EXPLORE:

While Watching the Video

The following focus questions help both teachers and curriculum leaders think about where students are in their development of the concept of exponential decay, and in their ability to reason algebraically. Each person should select one or two questions to focus on while watching the video.

Focus Questions

- What is the evidence that students are engaged?
- What is the evidence that students expect to make sense and that they hold each other accountable for making sense?
- Was enough included in the Launch to get students engaged in the problem? Too much?
- Do students do what you predicted in the Explore phase?
- What does the teacher do during the Explore phase?
- Can you discern a purpose for the questions and comments the teacher makes during the Explore?
- Does anything unexpected happen? How does the teacher deal with it?
- Is the selection and sequencing of student work in the Summary phase effective? Are there unexpected student comments? How would you deal with these?
- Does the class make connections among solutions?

Form Focus Groups of Teachers and Teacher Leaders

It has worked well in the past to re-arrange participants into focus groups before viewing the video. If they have a few minutes to talk about the focus question *before* the video and then time to debrief in small groups *after* the video I have noticed that the discussions are more coherent. I have tried to think of follow up questions that will help participants extend their thinking.

SUMMARIZE: Focus group Discussion after viewing the video	**Focus Questions (as above)**	**In Previous Workshops Teachers have said**	**Follow Up Questions**
Note: Alternative ways to conduct discussions: It can be unnecessarily repetitive if the same discuss/ view/discuss format is followed in every pd session. I have tried different formats. Some of these are described in the appendix.	• What is the evidence that students are engaged?	- *They are talking to each other about the mathematics* - *Some of them saw connections to a prior unit and went back to their files to make comparisons.*	- *Did you see the teacher do anything that would encourage students to take ownership of the mathematics?*[Ω]
	• What is the evidence that students expect to make sense and that they hold each other accountable for making sense?	- *They question and correct each other, for example, when one boy thought the graph would "go into the negatives."* - *They made a definition for decay factor in their own words and checked on suggestions.*	- *Was their student definition of decay factor correct?*[Ω]
	• Was enough included in the Launch to get students engaged in the problem? Too much?	- *I liked that the teacher picked up on student thinking about a prior problem in her introduction.*	

[Ω] She does not immediately give information, she refers students to each other, and she is always asking whether a strategy or expression makes sense.

[Ω] A *factor* is part of a product. When students write $64 \div 2^x$ they are not writing a product. But at this stage Kathy judged it is more important to have students take ownership than to be a stickler for accuracy. Eventually we would want students to see that one equation, $y = a(b)^x$, fits both exponential growth and decay, and in both cases *b* is a *factor*. Letting students write their own definition also brought up the idea of equivalence and how multiplying by a fraction can be rewritten as dividing by a different quantity.

Focus Questions (cont'd)	**In Previous Workshops Teachers have said**	**Follow Up Questions**
• Do students do as you predicted in the Explore?	- *They came up with some of the equations we predicted.*	
• Can you discern a purpose for the questions and comments the teacher makes during the Explore?	- *Some of her comments are to make students talk to each other, not to or through her.* - *When she asks, "Why does that make sense?" I think she's modeling what she wants students to do.*	-
• Does anything unexpected happen? How does the teacher deal with it?	- *The comparison with inverse variation surprised me. They looked up prior work!*	- *What might you do if students come up with a solution or a connection that surprises you?*[Ω]

[Ω] Kathy talks in her reflection about how surprised she was when several students saw that the graph of an inverse variation is similar to the graph of an exponential decay relationship. She went home and thought about it, and the next day she asked one of the students who had looked back at prior work what he had found out. This led to 2 conjectures, one correct and one wrong. But even the wrong one led to students making sense of why there would be no x-intercept in the exponential decay graph. Meanwhile another bonus for following up on this unexpected connection was that Kathy was able to help students see that the graphs may look similar but the underlying relationship between variables is not the same. Teachers cannot be prepared for all eventualities. Following up on an idea after having more time to reflect on its relevance and usefulness works well.

	Focus Questions (cont'd)	In Previous Workshops Teachers have said	Follow Up Questions
	• Is the selection and sequencing of student solutions in the Summary effective? Are there unexpected student comments?	- *I can see that if I had not predicted the multiple versions of the equation I would feel flustered.*	- *Are all 3 equations correct? Is it effective to call on three different student solutions in this order?*[Ω] - *How might Heather have come up with $y = 64(2^{-x})$?*
	• Does the class make connections among solutions?	- *They connect two versions of the equation: multiplying by a fraction or dividing by a whole number.*	
FINAL SUMMARY: Large group discussion after viewing the video	After participants have had an opportunity to talk in their small groups we should have a large group discussion. This gives me an opportunity to ask Follow Up questions as needed.		

[Ω] It's interesting to me that the students came up with 3 equations, and were comfortable enough with 2 of them to want to incorporate them in their spontaneous discussion of the definition for *decay factor*. Because the teacher accepted both versions of the equation the conversation became richer, as students tried to work out if the "factor" could be a negative. They finally decided that a *decay factor* was a whole number one divided by or a fraction between 0 and 1 that one multiplied by, to get the desired pattern of decrease in the table. (The teacher predicts, correctly, that students will leave the idea of dividing by a whole number behind, when they see that some decay rates do not easily translate into division.) Creating definitions and keeping records are norms in this classroom.

Kathy called on the girl with the "division" version of the equation first, maybe because several students had found this equation. She calls on the girl with the "negative exponent" version last. This lets students use their tables to compare this version to the equations they were able to make sense of. The class can make no sense of the version with the negative exponent, though they do revisit negative exponents several days later. The idea of equivalent equations is a bonus of having 3 solutions.