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"Making Sense of Symbols: Student Discourse (*Say It With Symbols*, 1.3)"

LAUNCH:
Before viewing the video of students doing *Say it With Symbols* 1.3

Getting Ready to View the Video: Plan to Teach 1.3.	**Before watching the video "Making Sense of Symbols: Student Discourse", which shows students talking about *Say it With Symbols* 1.3, participants should familiarize themselves with the problem as a first step in planning to teach the lesson.** We do not see on the video what the teacher did with the student work (except for a very brief beginning on the Summary). So we can decide for ourselves which student products we might select and how we might sequence them, and what questions we might ask to push students to clarify their thinking. These are also crucial parts of planning and have to be done on the spot, during the explore and summarize phases of a lesson. My instructions to teachers will be to do Problem 1.3 in small groups, anticipate likely responses and errors, explicate the mathematical goals of the problem, and make plans for what they will look for and ask about in the Explore phase. They should use the TE as needed. All of this is in anticipation of preparing to select and sequence the real student work that appears on the video. Some "teacher" questions may help foreshadow the video.

Discussion after participants have planned for 1.3	**Possible "Teacher" Discussion Questions:**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
	• Is there more than one way to answer the question? • What are some likely responses or errors?		- For each error you anticipate try to explain what exactly the student might not understand.
	• What will students learn just by doing the problem?		- Is *equivalence* involved in this problem? - In what way are students making sense of symbols in this problem?

Possible “Teacher” Discussion Questions:	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
• What will you look for in the Explore phase? What will you push for in the Summary phase?	- Different solutions. Errors. - I want students to be able to “read” the expression given, and connect it to a geometric context of their own creation.	- What might you ask students to help them “read” the expression given? - Will you push for equivalent geometric solutions (same area, different configuration)?
• How is this problem connected to the previous problem? How does it advance the previous Problem?	- Problem 1.1 was about writing equivalent expressions to fit a context. Problem 1.3 is about interpreting a symbolic expression.	- Can the given expression be interpreted differently?[Ω]

[Ω] The “x^2” term might be a square area with x units on each side. But it might be rectangular, with length 2x and width 0.5x, for example; dimensions may be adjusted to make the area x^2 square units without using x units for each side.

VIDEO: "Making Sense of Symbols: Student Discourse (*Say it With Symbols* 1.3)"

10 minutes, 9 chapters

Note: This video was edited to focus on students talking to each other. There are transcripts available to help teachers interpret what students say and assess their understanding.

EXPLORE While watching the video:

Note: An alternative that has worked well is to use a rubric to analyze discourse. See K. Hufferd-Ackles, 2004, in the Appendix. One drawback of such a rubric is that specific mathematics is not mentioned. However, teachers can compensate for this by referencing specific mathematics in the evidence/ examples they cite.

While watching the video teachers should keep their own lesson plans in mind and relate these to what they hear and see students saying and doing. It is difficult to teach when all the variables are not under your control. You may have a very good and full plan, but students may need more in the launch than you anticipated; or the explore phase may produce many solutions with unexpected errors, some of which you have never seen before; or there may be correct solutions you had not anticipated; or the goal of the lesson may seem to have got lost in the explore phase. Planning is part of your defense against these challenges; but you need to be flexible in executing your plan. Closely *listening to students*, fully understanding the mathematics and potential connections yourself, and keeping the overall goal in mind are your life rafts in these stormy seas. I think I will refer participants to 5 practices they can keep in mind while they execute a plan: anticipating likely responses (they already did this in the planning phase above); monitoring in the explore phase (we get to do a little of this on the video – but we can not actually ask students questions); purposefully selecting student work (we see the student work on the board, and we can decide what we want to do with this); purposefully sequencing student work; connecting student responses. The summary has to go beyond sharing to be effective. [Ω]

The following questions relate these practices to this video. Participants should choose 1 or 2 questions to focus on.

Focus Questions

- For each student you see on the video decide if the solution is correct or not. If you were monitoring this explore phase what might you ask? What is the purpose of your question?
- How are student solutions connected, if at all?
- After watching the students explore, and the beginning of a summary, what do you think students understand? What misunderstandings are there? How do their understandings/misunderstanding relate to your goals?
- How would you choose to continue the summary the next day?

[Ω] I made a checklist using terminology from "Orchestrating Challenging Tasks" (Smith et al) to help keep these 5 practices in mind. See Appendix.

Brief Focus Group Discussion	As always, if I have time, then placing participants in focus groups before the video gives then time to clarify the question, and makes the viewing time more productive. I have tried to think of follow up questions to use in the small and large group discussions after viewing the video.

SUMMARIZE: Focus Group Discussion after watching the video.	**Focus Questions (as above)**	**In Previous Workshops Teachers have said**	**Follow Up Questions**
Note: Alternative ways to conduct discussions: It can be unnecessarily repetitive if the same discuss/ view/discuss format is followed in every pd session. I have tried different formats. Some of these are described in the appendix.	• For each student decide if the solution is correct or not. If you were observing this in real time what might you ask? What is the purpose of your question?		- Was your question intended to help clarify students' thinking, assess understanding, probe into an error, extend thinking, or scaffold the problem to help a group explore effectively? - John 2's group has a correct solution. Are all the members of the group confident about this solution? What would you ask to probe or extend their thinking?[Ω]

[Ω] John 2's group has a completely correct solution; John 2 even indicates the comparison between the outdoor square and the indoor rectangle. Logan appears to understand this solution. I would want to ask the other members of the group to talk about the solution, because it looks like John 2 is the mathematical leader of the group. As for John 2 and Logan, I think I would ask if the "x^2" term has to represent a square x•x area. The previous day John 2 had suggested that we could re-write 4x as 16x(1/4) etc., so he is ready to explore manipulation of the symbols. If they don't have an answer for my question I could scaffold this by asking what the dimensions of a rectangle might be if the area is 4 square units. Does it have to be a 2 by 2 square? 4 by 1 rectangle? How many rectangles are possible? This may lead them to consider expressions equivalent to x•x.

Focus Questions cont'd (as above)	**In Previous Workshops Teachers have said**	**Follow Up Questions**
• How are solutions connected, if at all?	- *Some solutions have the same error, not recognizing that $\pi x^2/4$ represents the area of a quarter circle, radius x.* - *Some have the same error, not recognizing that the x used to represent half the length of a side of the rectangle, on one part of the drawing, has to have the same value when used to represent the side of a square, on another part of the drawing.*	- What are the similarities and differences that Olivia sees in the two groups of solutions? - What can we ask students to bring out connections? - How is Audrey's solution like/unlike Hailey's? What would you like to ask Audrey or Hailey?[Ω]

[Ω] I think that Hailey and Audrey can "read" the given symbolic expression as a sum of areas, but they are inconsistent in their use of a value for "x." However, their errors stem from two different roots: Hailey draws the outdoor pool correctly, but when she is told by her partner that she has to include the indoor pool on the final drawing she attaches the two without attempting to label any side lengths. Maybe if we asked Hailey to label all the sides of her drawing she would self-correct. Later she criticizes other solutions because they do not include a quarter circle – and she refers to the $\pi x^2/2$ and the $\pi x^2/4$ as a half and quarter circle, without seeming to recognize that the x would have to be the same measure of radius in both cases. Maybe asking her what the relationship should be between the areas of the half and quarter circle would cause her to think about her drawing again. Audrey's drawing shows the correct relationship between the half and quarter circle, but not between the "x^2" square and the "$8x^2$" rectangle. She seems to see this when she and her partner are labeling the drawing. But gets distracted when her partner talks about the parts of a circle. Maybe if we pushed her to complete the labeling she would have found her own error.

Focus Questions cont'd (as above)	**In Previous Workshops Teachers have said**	**Follow Up Questions**
• After watching the students explore, and the beginning of a summary, what do you think students understand? Misunderstand? How do the understandings/ misunderstandings relate to your goals for the problem?		- Why do you think that John thinks that Olivia's "x^2" square is not big enough? [Ω] - Is Hailey's initial drawing of the outdoor part of the pool correct? What is the error she makes in coming up with her final drawing? What question would you like to ask her?

[Ω] There seems to be a hierarchy of difficulty in working with symbolic expressions. It appears to be less difficult to find an expression to fit a context than it is to find a context to fit an expression. In 1.3 students are asked to find an area configuration to fit a symbolic expression. The question is scaffolded by giving part of the area configuration. The challenge is to look at the geometric drawing given, write symbolic expressions for areas of parts of the figure, identify the parts of the given symbolic expression that are not accounted for, and then add a geometric figure that would represent the "extra" symbolic expressions. Students on this video are at all points in the continuum in being able to carry this out. Some do not "see" areas when they look at the symbolic expression, though they could probably figure areas of a given rectangle or semi-circle. These students may think of "πx^2" as instructions for a computation, rather than a representation of an area. Some "see" areas when they look at the symbolic expression and can correctly ascribe terms to given parts of the geometric figure, but they inconsistently use the information to create the missing area. And some can complete the task correctly and even re-configure the area in different ways. I think that John 1, who thinks that Olivia's x^2 is not big enough is at the first level. His drawing showed a completely symmetric indoor-outdoor pool. I don't think he understood that the symbolic expression could be "read" to guide the shape and size of the missing part.

	Focus Questions cont'd (as above)	**In Previous Workshops Teachers have said**	**Follow Up Questions**
	• How would you choose to continue the summary the next day?		- Hailey focuses on the quarter circle for her criticism of the group of posters on the right. Would you want to ask about other similarities and differences? What do you think other students are doing while Olivia and Hailey make their comparisons? How do we keep them involved?
FINAL SUMMARY: Large group discussion after viewing the video	Each small group should have time to discuss their question(s) and then report to the large group. I need to listen closely to the comments at this stage so I can keep the discussion relevant to the topic of using student discourse/ student work in *making instructional plans.*[Ω] **One important question to add to the final summary is, "If you are Kathy (the teacher) viewing this video, what notes might you make to yourself about how you might alter your plans for the next time you teach this lesson?"** My hope is that this brings us to a point where participants realize that being a teacher is also about being a learner.		

[Ω] One of the things that comes up when teachers view this video is how to establish a classroom where students independently explore, challenge each other in small groups, or in the summary, and expect to give reasons for solutions or for challenges. There are several articles about this (see Appendix), and everyone who is successful seems to talk about modeling productive behavior. I have no one-size-fits-all answer for this question. If I am using this video with curriculum leaders we can talk about what it would take to establish a learning community *among teachers* so that they can support each other as they move towards the kind of "classroom participation structure" (Lampert, 2004), or "math-talk learning community" (Huffer-Ackles, 2004) they desire. (See Appendix)