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Transcript for October 26 – 27, 2007

"Reasoning about Exponent Patterns:
Growing, Growing, Growing, 5.1 - 2"

The class is working on Problem 5.1 - 2, "Predicting the Ones Digit" and "Operating with Exponents"

The video was shot in real time and edited from,
1.5 class days, to approximately 25 minutes.

Reasoning about Exponent Patterns: *Growing, Growing, Growing 5.1*
Class: 8th grade
Date: October 26 - 27
Real Time: 1.5 class days.
Edited to 25 minutes.

Chapter 1: Introduction
Time: Approximately 00:00 – 00:02:25 (times from start of video)

Title Slide

Slide
Launch
Real Time: 8 minutes

Slide
Problem 5.1 from text

Line 1: 00:00:20 T: You'll see a table like this. And in a moment you're going to get a table similar to that. And this is a powers table, so that we can take a moment and look at numbers and the pattern of numbers as they're growing exponentially. How this table works. Here you have your base numbers across the top. Over here you have the exponent that you're going to

Line 10, 00:00:46 raise that base number to. So for example, for this cell, I would have the base of 2 raised to the exponent 1, and they told me in standard form that's 2.

S: What? I don't get it.
Collin: Oh so it's like a multiplication table but with exponents.

T: Yeah. For this cell, you have a base of 2 raised to the exponent 4. So it was 2 to the 4th, which they said in standard

Line 20, 00:01:19 form is 16, 2 times 2 times 2 times 2. Here, the base was 2, I don't know which one I was at here, with an exponent of 8, so that was 2 to the 8th, and in standard form it's 256. So if I was going to fill in - oh, I'll just randomly pick a cell this one, how would I fill in that cell of this table?

Student: Oh, I know.

T: Jake.

Line 30, 00:01:53 Jake: 5 - it would be 5 to the 5th power.

T: Base of 5 raised to the 5th power. And I'd find that in standard form. What would this cell be? Ashley.

Ashley: 2 to the 7th power.

Collin: Kind of like the multiplication table except this one it does matter which number is first.

Line 40, 00:02:16	T: It does, because these are your base. So if I go back to this one, what would you be entering in your calculator to put in this cell? Collin: 7 to the 2. T: Base of 7 to the 2nd power -

Chapter 2: Initial Exploration of Table
Time: Approximately 00:02:26 – 00:03:20 (times from start of video)

Slide
The teacher assigns some individual exploration time to get the table filled out.

Real time: 5 minutes

Line 1, 00:02:56	Sarah: Okay, like you take this and multiply it by that, then you get that. And if you take this, er this and multiply it, then you get that.
	T: Ah, remember that, 'cause that's the type of thing we're going to look for in a minute. After it's filled in I'm going to ask you to look for patterns, and you've already found one. Great idea.

Chapter 3: Re-launch
Time: Approximately 00:03:21 – 00:06:08 (times from start of video)

Slide
New Launch
3 minutes

Line 1, 00:03:30 T: What I'm going to ask your group to do on that large sheet of paper that I passed out is look for patterns you see in this table. And while you were working on filling that out, there were probably some patterns that started to emerge. For example, when I was filling it out, I had gone across when I was doing it, and I don't know why, but this number, 729,
Line 10, 00:03:55 stuck out in my mind, because I hadn't expected 729, I guess. And then later when I was working on the chart, I saw it again over here, and that surprised me. So one of the first things that I happened to notice is that there were some numbers that appeared twice. So something that I would write down on my big sheet of paper would be, let's see, this is 9 to the 3rd, and this is 3 to the 6th. So on my big
Line 20, 00:04:25 sheet of paper one thing that I noticed is that 9 to the 3rd is equal to 3 to the 6th. Sarah, when you very, when you very beginning to fill this out, you saw a pattern. What was the pattern you saw right away?

Sarah: I saw that like, going down, like at first when I started doing the 3's, I did the first row with the exponent as 1, and then when I did 9, er, uh, 3 with the
Line 30, 00:04:54 exponent of 2, I saw that it was 9 and it multiplied by 3, so then I tried, uh, 3 to the 3rd power and I got 27, so I knew that it multiplied by 3 each time and it worked for all the other ones just using the base.

T: So you're saying if you knew this one, you could just go this times this, and you'd end up there?

Sarah: Yeah.

Line 40, 00:05:16 T: And this times this and you'd end up there?

Sarah: Like 27 times 3.

T: So you could go back to here?

Sarah: Yeah.

Line 50, 00:05:44

T: So I could write down what she's saying with exponents and say, "Well, gee, that tells me 3 to the 1st times 3 to the 2nd is going to give you 3 to the 3rd." And she also noticed that 3 to the 1st times 3 to the 3rd was going to give her 3 to the 4th. And there's all sorts of patterns in here. Maybe there's some patterns going across. Maybe there's some patterns going up and down. Maybe you've seen some other patterns. I want your group after you have your whole powers table there to fill up that big sheet of paper with as many patterns as you can find on this powers table with these numbers.

Chapter 4: New Explore: Searching for Patterns
Time: Approximately 00:06:08 – 00:10:50 (times from start of video)

Slide
New Explore
Real Time: 20 minutes

Line 1, 00:06:20 T: So your idea of continuing to multiply by that number is working.

James: Yeah.
T: But somehow when you were doing it before it wasn't working out.

James: I've got - I crumpled it up┼

Logan: Whoa. I found something already.

Logan: Better go get a marker.

(new group)

T: I do like that the numbers are too big,
Line 10, 00:06:37 though. When did the numbers get too big?

Kendra: Around the 5.

Carson: When they got to like six numbers.

T: Did it bother you when the numbers got big?

Carson: Yes, because I couldn't fit'em in the box.
T: Did it surprise you that they got big?

Carson: Actually, yeah, because I still think that sometimes like 9 times 8 ...

Line 20, 00:06:54 T: So you're still trying to break yourself of it's not 9 times 8, it's 9 times 9 times 9 time -

Carson: I do it right, I just still think -

T: Still feel it. So by having to squish those numbers in there, it's forcing you to feel it.

T: Well, where's a million? Can you find a million? So if that one's a million, what would this one be?

Line 30, 00:07:13 Carson: A billion.

T: So right after one million is a billion?

Carson: I don't know. Oh, ten, ten million.

T: So what is this one?

Carson: A hundred million?
T: I think it is.

(new group)

Sarah: 9 times 81 equals 729.

Mike: Yeah, see.

Line 40, 00:07:32 Jake: The numbers up top, the numbers up top times 2 equal their, like, double what they are, or, wait -

Mike: Well, you have to - to their exponent

Sarah: 3 to the 6th? Is that right? 3 to the 6th?

Mike: Yeah, like, yeah -

Sarah: So you can do almost any of them?

Mike: Yeah. If you like - there's a
Line 50, 00:07:54 pattern with like skipping - like if you, if you get that one, like 3 to the 2nd times, like, 3 to the

Sarah: 4th.
Mike: 4th equals like 3 to the 6th.

T: What do you mean by skipping?

Mike: Like it skips an exponent, like, instead of going to 3 to the 3rd, you go to 3 to the 4th, and then you go to 3 to the 6th. So you skip 3 to the 5th. And
Line 60, 00:08:18 when you multiply those together it equals, it equals 3 to the 6th.

T: What if you did 3 to the 2nd times 3 to the 3rd, and you didn't skip one?

Mike: It would equal 243.

T: You think it would?

Sarah: 3 to the, alright, 3 to the 2nd is 9, and 3 to the 3rd is 27 - 9 times 27 equals 243.

Mike: Yup.

Line 70, 00:08:41 T: So you were right.

Sarah: But what I don't get is like, how do you know, 'cause like 2 times 4 isn't 6, it's 8.

(teacher leaves)

Sarah: Like, it's like an equation or something,that we can say like why is it doing this?

Jake: Well, I don't know if it'd be a correct equation.

Mike: What about like, what about like 3
Line 80, 00:09:02 to the 7th divided by 3 to the 5th would equal like 3 to the 3rd. Would that work?

Sarah: So wait. 3 to the 7th divided by 3 to the 5th -

Mike: 3 to the 7th

Sarah: Says 9.

Mike: Divided by 3 to the 5th equals -
Sarah: 3 to the 7th - it equals the 2nd.

Sarah: It equals the 2nd. I'm writing that stinker down.

Line 90, 00:09:28 Jake: Basically taking away that -

Mike: Yeah, taking away -

Jake: Taking away 2.

Mike: Taking out 2.

Jake: So division is basically subtraction.

Mike: Wait, sort of, yeah.

Sarah: No, that's not right.

Jake: The exponents are subtracting.

Mike: Yeah.

Line 100, 00:09:52 Sarah: Wait. Here they're adding. 2 plus 5 is 7, 2 plus 3 is 5, 2 plus 4 is six, 2 plus 3 is 5, 2 plus 4 is 6, 1 and 4 is 5.

Jake: Subtracting -

Mike: Yeah, it's subtracting, so write that down.

Sarah: 1 plus 4 equals 5, 2 plus 1 equals 3, 2 plus 4 equals 6, 2 plus 3 equals 5, 2 plus 4 equals 6, 2 plus 3 equals 5, 2
Line 110, 00:10:30 plus 5 equals seven, and 7 minus 5 equals 2. We finally got it, I think.

Mike: Wait. So when you, so when you multiply it's really, it's, it's adding -

Sarah: Adding exponents.
Mike: And when you divide, it's really just subtracting exponents -
Sarah: Subtracting exponents.

Chapter 5: Teacher’s Reflection after Exploration 5.1
Time: Approximately 00:10:51 – 00:12:54 (times from start of video)

Slide
The summary for 5.1 occurs the next day. Kathy explains in her reflection how she planned for this Summary.

Real time: 50 minutes

(Transcript for teacher reflecting is not available)

Chapter 6: Students Reasoning about Powers
Time: Approximately 00:12:55 – 00:16:10 (times from start of video)

Slide
End of Summary, October 27

Students Reasoning about the Rule
$(a^m)^n = a^{mn}$

Line 1, 00:13:06 S: 2nd equals 3 to the 4th.

T: And just double check - is that true on your powers table? Are 9 to the 2nd and 3 to the 4th equivalent?

S: Yup.

T: Are they both 81?

S: Yeah.

T: Sorry, go ahead.

Line 10, 00:13:24 Becca: So then we did 3 times 3 times 3 times 3, which is that just written out, and then we noticed that this equals 9, and then this equals 9, and so then that would equal 3 to the 2nd times 2, because we have two of the 3 to the 2nd, which is equivalent to 9 to the 2nd.

T: So you could see you have two 9's in here, and then she rewrote it as this is 3 to the 2nd, squared. Can you show us
Line 20, 00:14:12 another one?

Becca: We had 4 to the 2nd equals 2 to the 4th.

T: So another one where we add something to the 2nd power and something to the 4th power. Okay.

Becca: And then we -
T: Does this work first, before she goes on? Are 4 to the 2nd and 2 to the 4th equivalent on your powers table?

Line 30, 00:14:32 Class: Yes.

T: Yes? Okay, go ahead, sorry.

Becca: Then we wrote it out as 2 times 2 times 2 times 2. And then the 2 times the 2 equals 4, that 2 times the 2 equals 4,

which is equal to 2 to the 2nd times 2, which is equivalent to 4 to the 2nd.

Jon: You've lost me..

Becca: Because right here it's 2 times 2 equals 4, so we have 2 times 2, which is
Line 40, 00:15:10 right here, and then we have two of those and you get that.

Jon: Oh, and then the 4 would be the 2, the 2 times the 2 in the parentheses. I get it.

Becca: We did, we did, we had 9 to the 3rd is equal to 3 to the 6th, and then, so that'd be 3 times 3 six and then we took the 3 times the 3 equals 9, 3 times the 3 equals 9, 3 times the 3 equals 9. And
Line 50, 00:15:54 then that would be 3 to the 2nd to the 2nd equals 9 to the 3rd oh, that's supposed to be a 3.

Chapter 7: The Teacher Refocuses the Class
Time: Approximately 00:16:11 – 00:21:58 (times from start of video)

Line 1, 00:16:12 T: I found it interesting when I was looking over their poster last night, that they took their exponential forms and then to prove that they were equivalent to each other, they wrote it out in expanded form, and then they rewrote their expanded form in a way we haven't seen before. So I was thinking, "What if I had written something like this?" (*Writes 2•2•2•2•2•2*)

Line 10, 00:16:37 In expanded form. And then I tried to group it in a way that they did over here.

T: And if I tried to group like they did, how could I write it in one of these ways to just look at it differently?

S: It would be 4 to the ...

T: Where's the 4 coming from?

S: 2 times 2.

T: So I've got 2 squared -

S: Yeah. Times 3.

Line 20, 00:17:10 T: And by the times 3 you mean I have to take that times itself three times.

S: Yes. Yeah.

T: So I take this to the 3rd power.

S: What?

T: 'Cause I have 2 squared times itself three times.

S: Which would be 4 to the 3rd power.

T: Which is also what I wrote originally was 2 to the 6th power.

Line 30, 00:17:33 S: Oh, can you do 4 to the 3rd?

S: Well, what -

S: Wouldn't it be the same as 4 to the 3rd, though?
S: 2 to the 6th power.

T: Which is also the same as 4 to the 3rd, but for a moment I was just looking at what

if I kept it as a base of 2. Could I look at the way they were writing all that and make some sense of it?

Line 40, 00:17:49 T: What if I had 4 times 4 - 2, 3, 4 - 1, 2, 3, 4, 5, 6, 7, 8 - so you guys would write that as 4 to the 8th power, right?

Class: Yeah.

T: So we know it's equivalent to 4 to the 8th power. If I try to group it the way Becca and Josh and Steven did, if I group it like this now how can I write it instead of 4 to the 8th? Jake.

Jake: 4 to the 2nd power, yeah, 4 to the
Line 50, 00:18:21 2nd power, parentheses around that, and then put 4 on, on the, uh, 4, exponent on the outside of that.

T: I have four groups of that which is the same as -

S: 4 to the 8th.
T: 4 to the 8th.

T: So what if I had - you guys would write that as 6 to the 4th, but now if I grouped it differently

Line 60, 00:18:47 Jon: Oh, I know... maybe not.
T: I could group it how? 6 to the 2nd

S: To the 2nd, parentheses to the -

T: Which is also 6 to the 4th.

S: I get it now.

T: Alyssa?

Alyssa: What if there is an odd number?

Alyssa: So you couldn't do it if there, if there was an odd number, it wouldn't work.

T: Well, depending on how I wanted to
Line 70, 00:19:13 group it. If I wanted to group it the way Becca's group was, with the groups of two, no, but maybe there's a way I could group it in groups of three.

Sarah: Yeah.

T: Can you try one, Sarah?

Sarah: If there were nine of them -

T: So nine 6's?

T: So now how do you want me to group this, Sarah?

Line 80, 00:19:30 Sarah: Three.

T: Groups of three?

Sarah: Yeah.

T: So now how could I write it?

Sarah: It'd be 6 to the 3rd parentheses, and then that to the 3rd.

T: Is that the same as 6 to the 9th?

Alyssa: I don't understand where you're getting like, where at the end it has the number and then, like 6 and then the 3rd
Line 90, 00:19:53 I don't understand how you get - yeah, that number?

T: How do we get that number out here?

T: Somebody explain to Alyssa how we get this number out here. Collin.

Collin: Because that's how many of the groups of the groups you have before.

T: So the 6 to the 3rd is telling me this this is 6 to the 3rd, this is 6 to the 3rd, this is 6 to the 3rd.

Line 100, 00:20:21 Collin: And then there's three of those.

T: We've always been looking at the groups, or expanded form in making the groups. What if somebody said I have this? What is that all cleaned up written as one base and an exponent? 4 to the third squared. How many 4's would I have written out altogether?

Class: Two. Six. Six.

T: Why would I have six 4's?

Line 110, 00:20:54 S: There's two groups of three 4's.

T: So in expanded form there's one 4 to the 3rd and another 4 to the 3rd. So I

could write this instead of 4 to the 3rd to the 2nd, I could write this as 4 to the -

Class: 6th.

T: 6th. If I had 6 to the 5th and that's all raised to the 3rd power, I could write that as 6 to the -

Class: 15th.

Line 120, 00:21:26 T: 15th.

Class: Oh, you multiply'em.

S: Multiply the small numbers.

James: You multiply - you multiply the exponents.

T: How could I write that on our list over there?

S: Uh, A to the M -

T: A to the M. All of that to the N - sorry, Heather, what did you say?

Line 130, 00:21:49 Heather: Uh, you could have A to the A times, uh, M times N.

T: M times N.

Chapter 8: Students Make Conjectures about Exponential Forms
Time: Approximately 00:21:59 – 00:23:22 (times from start of video)

Line 1, 00:21:59 Student: What if there is a leftover? Is there a way you can incorporate that in the end?

T: So Elle, tell me again what it is you wanna do here.

Elle: Umm, How 'bout we make... How many is there?

T: I think I have 8 of 'em up here. Is that how many you want? Or do you want 9?

Line 10, 00:22:17

Elle: 9.
T: 9.

Elle: So make - Take 8 of those and make two groups. And then just have the last one leftover.

T: So those two groups would be what?

Class: 6 to the - 6 to the 4th.
T: 6 to the 4th -

S: the 2.

T: And all of that to the 2, but what do
Line 20, 00:22:36 I - what does this 6 have to -

S: Times 6.
T: Times 6.

Class: Aaah, oooh, I see. You called that one.

Elle: Okay, what if you have like, okay, if you had those eight back like instead of nine, and like erased the lines and stuff - thanks - and um, you had a group of three, and then another group of three,
Line 30, 00:22:58 and then you had one with just two in it.

T: Well, we can definitely write about this one. That would be what? 6 to the 3rd to the 2 -

S: Times 6 to the 2nd power.

T: Times 6 to the 2nd power. Or just 36.

Logan: I got it. That's tricky.

Chapter 9: The Teacher Looks Ahead
Approximate Time: 00:22:22 – 00:25:04(times from start of video)

Transcript not available