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"Developing an Algorithm for Multiplying Decimals: Bits and Pieces III, Problem 2.1"

LAUNCH:
Before viewing the video of students doing *Bits and Pieces III*, 2.1.

Before viewing the video participants need to do *Bits and Pieces III*, 2.1. I think I will launch 2.1 by having a "teacher" discussion. In the Getting Ready a strategy for multiplying decimals is demonstrated. I can ask participants what answer they would give for "0.3 x 2.3." Could they have estimated the product so they could check the reasonableness of their solution? Where and when did they acquire their algorithm? Why does it make sense? Based on my past experience I can predict that participants will have difficulty "making sense" of their computational process. The point of having them do 2.1 as if they were students is to explore "making sense" of multiplying two decimals, by estimating and by thinking of the decimals as fractions.

After the "student" summary of 2.1, I need to prepare participants to watch the video. If my audience is mostly teachers then my questions should focus mostly on the mathematical development and the teacher's role; if my audience is mostly curriculum leaders then my questions should focus on wider issues such as the reasons for having students take the time to develop an algorithm, the evidence that students retain prior learning, and the need to view the curriculum as a connected whole.

Getting Ready to View the Video	**Possible "Teacher" Discussion Questions:**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
After participants have done 2.1	• What would be your goal for 2.1? (We can refer to the TE.)	• *Connect place value of digits in decimal product to denominator of fraction product* • *Make sensible estimates*	- *Do the steps in multiplying the fraction forms connect to your algorithm for multiplying decimals?* - *Are estimates useful?*

Possible "Teacher" Discussion Questions cont'd:	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
• What skills do you expect students to have to use to follow the instructions on page 22 of the student text?	• *They have to rewrite decimals as fractions and use their algorithm for multiplying fractions.* • *They have to make sense of the sizes of numbers.*	- *What do you mean "make sense of numbers?" Is this "number sense?"*[Ω]
• What difficulties might you predict for your students in 2.1?	• *They might have forgotten how to multiply fractions* • *They might have trouble placing the decimal point when they change from an improper fraction to a decimal*	- *What might help students place the decimal point correctly?*
• What do they still have to understand after doing 2.1?	• *They will have a process for multiplying – change to fraction, multiply, change back. But this is not as efficient as the standard algorithm*	- *How does the standard algorithm relate to what the students are doing?*

[Ω] In doing Problem 2.1 students have to make sense of the sizes of the quantities as they move back and forth among representations. They must understand the meanings embodied in place value, in denominators, and in computation symbols.

Possible "Teacher" Discussion Questions cont'd:	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
• How does fraction multiplication connect/interfere with student understanding of decimal multiplication?	• *Multiplying the numerators of the fractions tells the digits in the answer. Multiplying the denominators tells the place values of the digits.*	
• Do you expect students to propose an algorithm at this stage?	• *Not the standard one* • *Some students may have met the standard algorithm in elementary school*	- *What are the pros and cons of students having met the algorithm in elementary school?*[Ω]

[Ω] Sometimes trying to get students (or adults) to make sense of something that they learned by rote is harder than getting them to make sense of a new idea.

VIDEO: "Developing an Algorithm for Multiplying Decimals"
(Bits and Pieces III, 2.1, 15 chapters. 29 mins)

Note: This part of the video has been edited to focus on the learning trajectory from first exposure to a procedure for multiplying decimals by switching to fractions, to the emergence of a student algorithm. Real time is 1.5 class periods.

EXPLORE: While watching the video

Principals benefit from doing the mathematics in 2.1, and having this in front of them as they watch the video, but I need to remember that their responsibility includes supporting teachers. When they are viewing the video they should be asking themselves one or more of the following:

Focus Questions for Principals

* What evidence is there of student engagement? Of students taking responsibility for learning?
* How is this way of teaching different from what teachers are currently doing? How prepared are the teachers in my school?
* How can I help teachers move towards inquiry based teaching?
* What is the responsibility of teachers in knowing what students bring with them from prior curricular experiences?
* What is my role in supporting this program?

Focus Questions for Teachers

Teachers will benefit from having available a copy of a transcript, perhaps 1 per pair of participants, and copies of 2.1. To Launch into viewing the video I should remind teachers of what they said their goal would be for 2.1, and what they expected from their students.

Each person should choose a focus question from the list below, and imagine Teri re-viewing the video from this perspective.

- What moments appear to be important mathematically? Was this a student interchange? What was the teacher's role?
- What is the teacher's role when the students are exploring? Are her actions and questions effective in advancing the mathematical goal of the lesson?
- What are the students doing during the "explore" phase? Are their actions and conversations effective in addressing the mathematics?
- What prior skills do you see students using? Are they using these correctly? Effectively?
- What is the teacher's role in the Summarize phase? How does her selection and discussion of student work advance the mathematical goals?
- What is the evidence that the students are learning?
- What evidence is there that students expect to make sense of their various conjectures and strategies? What is the teacher's role in creating and raising these expectations?
- Where are students on the path towards making sense of an efficient algorithm for multiplying decimals?

Form Focus Groups of Teachers and Teacher Leaders

It has worked well in the past to re-arrange participants into focus groups before viewing the video. If they have a few minutes to talk about the focus question *before* the video and then time to debrief in small groups *after* the video I have noticed that the discussions are more coherent.

I have noticed a tendency for people to make general comments in response to the video. When I ask follow up questions both the small and large group discussions are richer.

	Focus Questions (as on previous page):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
SUMMARIZE **Focus Group Discussion after Viewing the Video**	• What moments appear to be important mathematically? Was this a student interchange? What was the teacher's role?	• *Nikki comes up with a pattern (chapter 13): multiply the digits as if they are whole numbers and then multiply the denominators and use that to place the point. Teri's response is to check her suggestion in other examples*	- *How is Nikki's suggestion like/unlike the algorithm you have?*
Note: Alternative ways to conduct discussions: It can be unnecessarily repetitive if the same discuss/ view/discuss format is followed in every pd session. I have tried different formats. Some of these are described in the appendix.	• What is the teacher's role when the students are exploring? Are her actions and questions effective in advancing the goal of the lesson?	• *She is helping, asking questions.* • *She keeps the focus on making sense of the answer*	- *What is Teri looking for in the Explore phase? How does this relate to the mathematical goal for 2.1? Can you plan ahead to make the Explore phase productive?* - *What are some questions she asks? What is the purpose of her question?*

Focus Questions (cont'd):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
• What are the students doing during the "explore" phase? Are their actions and conversations effective in addressing the mathematics?	• *They are having good discussions about whether their answers make sense.* • *One group is not successful remembering or looking up an algorithm for multiplying mixed numbers.*	- *The teacher was not present when Allie's group used an incorrect algorithm for multiplying mixed numbers (Chapter 6). If she had been what might she have done?*
• What prior skills do you see students using? Are they using these correctly? Effectively?	• *Converting fractions to decimals and vice versa.* • *Multiplying fractions.* • *Rounding* • *Estimating size by using benchmarks*	- *As a piece of practice how many fraction multiplications did students do in this lesson? Whole number multiplications?*
• What is the teacher's role in the Summarize phase? How does her selection and discussion of student work advance the mathematical goals?	• *She chooses student work to share.* • *She writes the fraction and decimal form next to each other and asks students to study each pair of sentences.*	- *Why doesn't the teacher end the summary when she gets a proposed algorithm from Nikki? What does she do with this?*[Ω] - *Is Teri's scripting of the number sentences effective? Why?*[Ω]

[Ω] The teacher checks Nikki's suggestion works in all examples so far, but she seems to think that students need more evidence for why this works, so she produces further examples so that students can continue to check on the connections between fraction and decimal multiplication. She must think it would be premature to close the thinking down.
[Ω] Teri's scripting of the number sentences puts ideas next to each other in ways that make patterns more obvious. She is not re-doing student work, just focusing it.

Focus Questions (cont'd):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
• What is the evidence that the students are learning?	• *They are engaged.* • *They question each other, explain to each other*	- *After viewing the video Teri had some concerns about the "tenths times tenths" language. What do you think about this? Can we turn this into a teaching moment?*[Ω]
• What evidence is there that students expect to make sense of their various conjectures and strategies? What is the teacher's role in creating and raising these expectations?	• *Students correct each other – like when Allie's group presents a wrong solution or when Ellie and Drew disagree (Ch.10)* • *Students question themselves – like when Nikki does not like Kaylie's answer of 22 because it's "not even close to our estimate.*	- *What does Teri ask most often?*[Ω] - *What are some outcomes of having a student group put an error on the board? Do you think the teacher planned for Allie's group to present an error?*[Ω] - *What classroom norms do you see in operation?* [Ω]

[Ω] Teri's concern is that meaning might be lost when she and students say "tenths times tenths" instead of "tenths of tenths," as in "tenths times tenths are hundredths." By shortcutting the language we may be shortcutting the thought process too. When we say "tenths of tenths are hundredths" we are verbalizing a process which gives meaning to the sizes of the factors and the product. When we say "tenths times tenths are hundredths" students might actually be thinking of "10 x 10 = 100" and not concentrating on meaning.

[Ω] A lot of Teri's questions are "Why do you do that?" (or some variant) or "Does that make sense?" By making these part of the classroom norm students seem to have internalized these. We hear them asking similar questions in small groups.

[Ω] I don't think Teri planned for that error. However, the error is a common one, and the result of sharing it was to review a procedure, for those who had forgotten. Most importantly it gave other students an opportunity to make a correction. It was noticeable that this seemed to be another classroom norm. There were no hurt feelings or disparaging remarks, just a businesslike correction.

[Ω] Asking questions, making sense, appreciating each others work, are all norms. For example, Jesse praises Nikki's suggestion, "Nikki's got it."

Focus Questions cont'd:	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
• Where are students on the path towards making sense of an efficient algorithm for multiplying decimals?	• *They have a routine, but it's not as fast as the standard algorithm. It makes sense, though.*	- *What is the connection between the routine students have arrived at and the standard algorithm? Would you push towards the standard algorithm?* [Ω]

[Ω] I think they are ready to make an algorithm that is *like* the standard in that it multiplies the "numerators" then replaces the decimal point by referring to place value equivalent to the product of denominators. I think Teri wants them to think more about "how much smaller the answer got" which gets us into thinking of the denominator as a divisor. For example, 0.2 x 0.3 is 100 times smaller than 2 x 3. Dividing 6 by 100 is equivalent to moving the decimal point 2 places. Students are not quite there yet.

	Focus Questions:	In Previous Workshops Principals Have Said:	Follow Up Questions:
Focus Questions for Principals	• What evidence is there of student engagement? Of students taking responsibility for learning?	• *Students are working independently of the teacher, discussing, explaining.* • *Students have notebooks and refer to notes.* • *Students correct the errors of other students.*	
	• How is this way of teaching different from what teachers are currently doing? • How prepared are my teachers?	• *The teacher has to be flexible because the direction of the class is not entirely under her control*	
	• What can I do to help teachers move towards inquiry based teaching?	• *I need the help of someone with mathematical expertise to focus pd meetings.*	
	• What is the responsibility of teachers in knowing what students bring with them from prior curricular experiences.	• *I can help them with this by scheduling regular cross grade meetings*	
	• What is my role in supporting this program after implementation?		- Is this way of teaching going to change your evaluation procedures?[Ω]

[Ω] Principals are busy people, so any classroom evaluation has to be efficient and effective. Having a common language to talk about what is observed helps both parties

Large Group Discussion after Viewing the Video

After participants have had an opportunity to talk in their small groups we should have a large group discussion. This is another opportunity to ask follow up questions.

The mathematics that Teri is trying to draw out is quite subtle. On the surface, counting decimal places and placing the point seems quite a simple procedure, but the ideas underlying this involve connecting fractions to decimals, in particular seeing the denominator as a divisor and reasoning proportionally about how that connects to placing the decimal point. For example, 0.2 x 0.3 is related to 2 x 3, but 2 is 10 times larger than 0.2 and 3 is 10 times larger than 0.3, so 2 x 3 is 100 times larger than 0.2 x 0.3. So, following the student idea of an algorithm at this stage, we would multiply 2 x 3 = 6 and then make the answer 100 times smaller, that is divide by 100, to get 0.06.

Students have not reached this understanding at this stage, but Teri is laying the groundwork, and I need to be able to expose what she is doing, by judiciously asking follow up questions. Teri's work on this video is an indication of her knowledge of the mathematics, her understanding of what her students know already, and her mastery of the pedagogy.

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to communicate about improvement. There are some articles about teaching practices which are compatible with the CMP approach to teaching and learning. It may help both teachers and Principals to read these and understand what is meant by: *predicting likely responses*, *selecting and sequencing* student work, *connecting* student work, f*unneling* or *focusing* questions. See Smith et al in the Appendix.